

Square Roots

Basic Concepts

- When a number is multiplied by itself, the resulting product is a *perfect square*.
- Therefore, that number is the **square root** of the perfect square.
- The symbol for square root, √, called a radical sign, denotes the principal or nonnegative square root. (Although a negative number multiplied by itself also results in a positive perfect square, principal square roots are nonnegative by definition.)
- The expression under the radical sign is called the radicand.
- Therefore: $\sqrt{25} = 5, \sqrt{4} = 2, \sqrt{100} = 10$
- The square root of a negative number is not a real number. No number squared equals a negative product. Example: √-25 has no real number solution.

However, $-\sqrt{25} = -5$, since the negative sign is outside the radical sign.

• A square root multiplied by itself is equal to the radicand. $\sqrt{4} \cdot \sqrt{4} = 2 \cdot 2 = 4$

1	_	1
2^2	=	4
3 ²	=	9
4 ²	=	16
5^2	=	25
6^2	=	36
7^2	=	49
8^2	=	64
9 ²	=	81
10^{2}	=	100
11 ²	=	121
12 ²	=	144
13 ²	=	169
14 ²	=	196
15 ²	=	225

Product Property of Square Roots

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

• Therefore, $\sqrt{4 \cdot 25} = \sqrt{100} = 10$ and $\sqrt{4}\sqrt{25} = 2 \cdot 5 = 10$

Simplifying Square Roots

- When the radicand is not a perfect square and does not have a factor that is a perfect square, then it is in *simplest radical form* and cannot be computed without a calculator.
- When the radicand is not a perfect square but has a factor that is a perfect square then it can be simplified by finding the square root of the perfect square factor and leaving the remaining factor as the radicand.

Example: $\sqrt{12} = \sqrt{3 \cdot 4}$ Since 4 is a perfect square, this can be simplified. $\sqrt{4} = 2$, therefore the simplest radical form is $2\sqrt{3}$.

• Variables with even exponents are always perfect squares.

Example: $\sqrt{x^2} = x$ because $x \bullet x = x^2$, $\sqrt{x^6} = x^3$ because $x^3 \bullet x^3 = x^6$. To solve, find the two equal powers that add up to the exponent in the radicand.

Variables with odd exponents are not perfect squares, but can be easily simplified.

Example: $\sqrt{x^3} = \sqrt{x^2} \bullet \sqrt{x} = x\sqrt{x}$, $\sqrt{x^{11}} = \sqrt{x^{10}} \bullet \sqrt{x} = x^5\sqrt{x}$ If the exponent in the radicand is odd, then subtract 1 from it. Simplify the perfect square factor and leave the remaining variable factor as the radicand.



Square Roots

Quotient Property of Square Roots and Rationalizing the Denominator

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}, b \neq 0$$

- Example: $\sqrt{\frac{100}{4}} = \frac{\sqrt{100}}{\sqrt{4}} = \frac{10}{2} = 5$ Example: $\sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{\sqrt{4}} = \frac{\sqrt{5}}{2}$ Note that $\sqrt{5}$ cannot be simplified
- Simplified radicals do not contain radicals in the denominator. In order to simplify, use a process called rationalizing the denominator, whereby the numerator and denominator are both multiplied by the radical denominator in order to eliminate it. Remember that a square root multiplied by itself is equal to the radicand.

Example: $\frac{2}{\sqrt{5}}$ To simplify, rationalize the denominator: $\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

Example: $\frac{3}{4+\sqrt{11}}$ To rationalize a denominator with two terms, multiply by the *conjugate* by changing the sign of the second term. Conjugates multiply as the difference of 2 squares, with the middle term dropping out.

$$\frac{3}{4+\sqrt{11}} \bullet \frac{4-\sqrt{11}}{4-\sqrt{11}} = \frac{3(4-\sqrt{11})}{16-4\sqrt{11}+4\sqrt{11}-11} = \frac{3(4-\sqrt{11})}{16-11} = \frac{3(4-\sqrt{11})}{5}$$

Rules for working with square roots when adding, subtracting, multiplying and dividing

- Simplify the square roots.
- Perform the indicated operations.
- Simplify the final answer.

Adding & Subtracting: Like square roots can be combined.

•
$$\sqrt{4} + \sqrt{9} = 2 + 3 = 5$$

•
$$\sqrt{12} + \sqrt{3} = \sqrt{4 \cdot 3} + \sqrt{3} = 2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$$

•
$$2\sqrt{5} - \sqrt{5} = \sqrt{5}$$

•
$$\sqrt{20} - \sqrt{5} = \sqrt{4 \cdot 5} - \sqrt{5} = 2\sqrt{5} - \sqrt{5} = \sqrt{5}$$

Multiplying & Dividing: Use the product and quotient rules as outlined earlier. Unlike square roots **can** be multiplied and divided. Simplify first, then perform the multiplication or division, simplify the answer.

• Examples:
$$\sqrt{16}\sqrt{36}=4$$
 • $6=24$, $\sqrt{6}\sqrt{24}=\sqrt{144}=12$, $\sqrt{3}\sqrt{5}=\sqrt{15}$

• Examples:
$$\sqrt{\frac{72}{6}} = \sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$$
, $\frac{\sqrt{48}}{\sqrt{3}} = \frac{\sqrt{16 \cdot 3}}{\sqrt{3}} = \frac{4\sqrt{3}}{\sqrt{3}} = 4$