



Concepts in Set Theory and Probability Theorems

Concepts in Set Theory

Set → A collection of well defined elements.

1. **Description** – A set defined in words.
Example: Set A is the set of Natural numbers ending in 10.
2. **Roster** – A set is defined with a list of elements surrounded by braces $\{\}$.
Example: $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
3. **Set Builder Notation** –
Example: $A = \{x | x \text{ is a natural number less than } 11\}$, which reads: "Set A is set of all elements x such that x is a natural number less than 11."

Element → An item in a set denoted by the symbol \in .

Example: If $A = \{1, 2, 3\}$, then $3 \in A$

Equal sets → are identical, containing exactly the same elements.

Example: If $A = \{A, B, C, D\}$, and $B = \{D, C, B, A\}$, then $A = B$

Equivalent sets → have the same cardinal number of elements, denoted by the symbol $n(\)$, but the elements do not need to be identical.

Example: If $A = \{1, 2, 3, 4\}$ and $B = \{\text{April, May, June, July}\}$, then $n(A) = n(B)$. Sets A and B are equivalent.

Empty or Null Set → is a set that contains no elements and are denoted by the symbols $\{\ }$ and \emptyset .

Subset → denoted by the symbol \subseteq occurs when all the elements of one set are also the elements of another. A subset may be, but doesn't have to be equal to the original set.

Example: If $A = \{A, B, C, D\}$ and $B = \{A, B, C, D, E, F, G\}$, then $A \subseteq B$.

Proper Subset → denoted by the symbol \subset occurs when the subset contains at least one less element than the original set.

Example: If $A = \{A, B, C, D\}$ and $B = \{A, B, D\}$, then $B \subset A$

Number of Subsets → is 2^n , where n is the number of elements in the set.

Example: $A = \{A, B, C, D\}$. Since set A has 4 elements, the formula for number of subsets is: $2^4 = 16$. Therefore, there are 16 subsets of set A. They are: \emptyset , $\{A\}$, $\{B\}$, $\{C\}$, $\{D\}$, $\{A, B\}$, $\{A, C\}$, $\{A, D\}$, $\{B, C\}$, $\{B, D\}$, $\{C, D\}$, $\{A, B, C\}$, $\{A, B, D\}$, $\{A, C, D\}$, $\{B, C, D\}$ and $\{A, B, C, D\}$. Note that the first fifteen subsets of set A are also proper subsets. The formula for the number of proper subsets is $2^n - 1$. In this example of set A, the number of proper subsets is $2^4 - 1 = 15$.



Concepts in Set Theory and Probability Theorems

Universal Set → contains all the elements for any specific discussion, and is symbolized by the symbol **U**.
Example: $U = \{A, E, I, O, U\}$

Intersection → contains the elements *common* to 2 or more sets and is denoted by the symbol, \cap .

Union → contains *all* the elements in two or more sets and is denoted by the symbol, \cup .

Complement → contains all the elements in the universal set that *are not* in the original set and is denoted by the symbol, $'$.

Example: $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 0\}$ $A = \{1, 2, 3\}$ $B = \{2, 3, 4, 5, 6\}$
 $A \cap B = \{2, 3\}$ $A \cup B = \{1, 2, 3, 4, 5, 6\}$ $A' = \{4, 5, 6, 7, 8, 9, 0\}$ $B' = \{1, 7, 8, 9, 0\}$

Probability Theorems

Complement → $P(E) = 1 - P(E')$
 $P(E') = 1 - P(E)$

Multiplication Rule → $P(A \cap B) = P(A) \times P(B)$
{AND, BUT, ALSO, AS WELL AS}

General Addition Rule → $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
{OR} Independent events, both events can occur

Special Addition Rule → $P(A \cup B) = P(A) + P(B)$
{OR} Events are mutually exclusive, and cannot occur simultaneously

Neither → $P(A \cup B)' = 1 - P(A \cup B)$
{NOT EITHER} Complement of the addition rule

Neither → $P(A \cup B)' = P(A') \cap P(B')$
{NOT EITHER} Only if independent events

Conditional → $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

{GIVEN THAT}