Radicals and Exponents

The basic properties of radicals are listed below.

$$\sqrt[r]{a^p} = a^{\frac{p}{r}} = \left(\sqrt[r]{a}\right)^p$$

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad [b \neq 0]$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

$$\sqrt[n]{a^n} = a$$

[if n is a positive even integer]

$$\sqrt[n]{a^n} = |a|$$

[if n is a positive odd integer]

$$\sqrt{a+b} = \sqrt{a+b} \text{ NOT } \sqrt{a} + \sqrt{b}$$

Simplest Radical Form

- No perfect roots under the radical sign

  Example: $\sqrt[3]{81}$

  $$\sqrt[3]{81} = \sqrt[3]{27 \cdot 3} = 3\sqrt[3]{3}$$

- No fractions under radical sign

  Example: $\sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{\sqrt{9}} = \frac{\sqrt{5}}{3}$

- No radical in denominator

  $$\frac{1}{\sqrt[3]{3}}$$

  $$\frac{1}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{9}}{\sqrt[3]{9}} = \frac{\sqrt[3]{9}}{\sqrt[3]{27}} = \frac{\sqrt[3]{9}}{3}$$

Radicals and Exponents

The basic properties of exponents are listed below.

$$a^r \times a^s = a^{r+s}$$

$$(ab)^r = a^r b^r$$

$$\left(a^r\right)^s = a^{rs}$$

$$\frac{a^r}{a^s} = a^{r-s}$$

$$\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$$

$$a^{-r} = \frac{1}{a^r}$$

$$a^{2r} = a^r \cdot a^r = a^{r+r} = \left(a^r\right)^2 \,\text{.}$$