Review Packet for Exam I - Math 130

1.) **Functions:**

Definition of a Function: A **function** is a correspondence between two sets of elements such that to each element in the first set there corresponds one and only one element in the second set. The first set is called the **domain** (possible values for x) and the set of corresponding elements in the second set is called the **range** (possible values for y). The **input values** for a function are from the **domain** and they are the **independent variables,** the **output values** for a function are from the **range** and they are the **dependent variables.**

Alternate definition for a function: If in an equation with two variables, we get exactly one output (value for the dependent variable) for each input (value for the independent variable), then the equation specifies a function

Vertical-Line Test: An equation specifies a function if each vertical line in the coordinate system passes through at most one point on the graph of the equation. If any vertical line passes through two or more points on the graph of an equation, then the equation does not specify a function.

Even Functions: $f(-x) = f(x)$ for example $f(x) = x^2$ then $f(3) = 9$ and $f(-3) = 9$

Odd Functions: $f(-x) = -f(x)$ for example $f(x) = x^3$ then $f(3) = 27$ and $f(-3) = -27$

Increasing Function: $f(x_1) < f(x_2)$ & $x_1 < x_2$

Decreasing Function: $f(x_1) > f(x_2)$ & $x_1 < x_2$

One-to-One Functions - a function f is said to be one-to-one if each range value corresponds to exactly one domain value. (horizontal line test)

Inverse of a function - The inverse of a function $f(x)$ can be found be replacing x's with y's and solving for y.

Sample Function Problems:

1.1 Find the domain of each function:

1.1.1 $f(x) = 2x^3 - x^2 + 3$ all real numbers or $\mathbb{R}$

1.1.2 $f(x) = \frac{x-2}{x+4}$ $(-\infty, -4) \cup (-4, \infty)$

1.1.3 $f(x) = \sqrt{7-x}$ $(-\infty, 7]$ or $x \leq 7$

1.1.4 $f(x) = \frac{1}{\sqrt{5+x}}$ $(-5, \infty)$ or $x > -5$

1.2 If $f(x)$ is an even function containing the point (4,2) then what other point must also be on the graph? $(-4, 2)$

1.3 If $f(x)$ is an odd function containing the point (-2,-7) then what other point must also be on the graph? $(2, 7)$

1.4 For each of the following functions determine if the function is increasing or decreasing :

1.4.1 $f(x) = 1 - x$ $\quad f(x) = -x + 1$

$m = -1$ negative slope, decreasing function

1.4.2 $f(x) = 3x+5$

$m = 3$ positive slope, increasing function

1.4.3 $f(x) = \frac{x}{2} - 3$

$m = \frac{1}{2}$ positive slope, increasing function

1.5 Find the inverse of the following functions:

1.5.1 $y = \frac{2x+1}{x-1}$

$x = \frac{2y+1}{y-1}$

$(y-1)x = 2y+1$

$xy - x = 2y+1$

$-2y + x \quad -2y + x$

$-2y + xy = x+1$

$y(-2+x) = x+1$

$\boxed{y = \frac{x+1}{x-2}}$

1.5.2 $4y = x^3 - 32$

$4x = y^3 - 32$

$4x + 32 = y^3$

$\boxed{y = \sqrt[3]{4x+32}}$

1.5.3 $3y = 2x - 7$

$3x = 2y - 7$

$3x + 7 = 2y$

$\boxed{y = \frac{3x+7}{2}}$

1.6 For $f(x) = 10 - 5x - x^2$, find:

a. $f(-4) = 10 - 5(-4) - (-4)^2 = 10 + 20 - 16 = \boxed{14}$

b. $f(2x) = 10 - 5(2x) - (2x)^2 = \boxed{10 - 10x - 4x^2}$

c. $f(x+h) = 10 - 5(x+h) - (x+h)^2 = 10 - 5x - 5h - (x^2 + 2hx + h^2)$

$= \boxed{10 - 5x - 5h - x^2 - 2hx - h^2}$

d. $4f(x) = 4(10 - 5x - x^2) = \boxed{40 - 20x - 4x^2}$

e. $\frac{f(x+h) - f(x)}{h} = \frac{10 - 5(x+h) - (x+h)^2 - (10 - 5x - x^2)}{h}$

$= \frac{\cancel{10} - \cancel{5x} - 5h - \cancel{x^2} - 2hx - h^2 - \cancel{10} + \cancel{5x} + \cancel{x^2}}{h}$

$= \frac{-5h - 2hx - h^2}{h} = \frac{\cancel{h}(-5 - 2x - h)}{\cancel{h}} = \boxed{-5 - 2x - h}$

2.) **Mathematical Models:** You should be able to define linear functions, polynomials, coefficients, degree of a polynomial, quadratic functions, cubic functions, power functions, exponential functions, and logarithmic functions.

Properties of Logarithms:

$\log_a MN = \log_a M + \log_a N$

$\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$

$\log_a M^r = r \log_a M$

$\log_a M = \frac{\log M}{\log a} = \frac{\ln M}{\ln a}$

Common Logarithms are logarithms with the base 10

Natural Logarithms are logarithms with base e

Sample Problems:

2.1 **Find x, y, or b without using a calculator:**

2.1.1 $\log_3 x = 2$ $\quad 3^2 = x \quad x = 9 \quad \log_3 9 = 2$

2.1.2 $\log_3(4x - 7) = 2$ $\quad 3^2 = 4x - 7 \quad 9 = 4x - 7 \quad 16 = 4x \quad x = 4$

2.1.3 $\log_4 x = \frac{1}{2}$ $\quad 4^{\frac{1}{2}} = x \quad \sqrt{4} = x \quad x = 2$

2.1.4 $\log_{\frac{1}{3}} 9 = y$ $\quad \frac{1}{3}^{y} = 9 \quad y = -2$

2.1.5 $\log_b 1000 = \frac{3}{2}$ $\quad b^{\frac{3}{2}} = 1000 \quad \sqrt{b^3} = 1000 \quad b = 100$

2.2 **Find x:**

2.2.1 $\log_b x = \frac{2}{3}\log_b 8 \div \frac{1}{2}\log_b 9 - \log_b 6$

$\log_b 8^{\frac{2}{3}} + \log_b 9^{\frac{1}{2}} - \log_b 6$

$\log_b 4 + \log_b 3 - \log_b 6$

$\log_b \left(\frac{4 \times 3}{6}\right) = \log_b 2 \quad \log_b x = \log_b 2$

$\boxed{x = 2}$

2.2.2 $\log_b x + \log_b(x - 4) = \log_b 21$

$\log_b x + \log_b (x-4) = \log_b 21$

$\log_b (x)(x-4) = \log_b 21$

$x^2 - 4x = 21$

$x^2 - 4x - 21 = 0$

$(x-7)(x+3) = 0 \quad \boxed{x = 7}$

can't take the log of a negative number

2.2.3 $2\log_3(x + 4) - \log_3 9 = 2$

$\log_3 (x+4)^2 - \log_3 9 = 2$

$\log_3 (x+4)^2 - 2 = 2$

$\log_3 (x+4)^2 = 4$

$3^4 = (x+4)^2 \quad 9^2 = (x+4)^2 \quad 9 = x+4 \quad \boxed{x = 5}$

3.) Linear Functions and Calculation of Slope (x_1, y_1) (x_2, y_2)

a. Find the slope of the line that contains the points (5,5) and (-3,1) m = $\frac{1}{2}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-5}{-3-5} = \frac{-4}{-8} = \frac{1}{2}$$

b. Find the equation of the line with slope = 3 which passes through (3,2). Put your answer in the slope-intercept form ($y = mx + b$). $y = 3x - 7$

$y = mx + b$ $\quad m = 3$

$2 = 3(3) + b$ $\quad b = -7$

$2 = 9 + b$

c. Find the equation of the line which passes through the points (2,-4) and (-1,2). $y = -2x$

$$m = \frac{2-(-4)}{-1-2} = \frac{6}{-3} = -2$$

$-4 = -2(2) + b$

$-4 = -4 + b$

$0 = b$

4.) Evaluate the limit, if it exists:

a. $\lim_{x \to 2} \left(\frac{9}{x-2}\right) =$ $\quad \frac{9}{2-2} = \frac{9}{0}$ limit does not exist

b. $\lim_{x \to -2} (6x^2 + 4x - 20) =$

$6(-2)^2 + 4(-2) - 20 = 24 - 8 - 20 = -4$

c. $\lim_{h \to 0} \left(\frac{((h-4)^2 - 16)}{h}\right) =$ $\quad \frac{(0-4)^2 - 16}{0} = \frac{0}{0}$ so, factor the numerator

$$\lim_{h \to 0} \frac{h^2 - 8h + 16 - 16}{h} = \lim_{h \to 0} \frac{\cancel{h}(h-8)}{\cancel{h}} = 0 - 8 = -8$$

d. $\lim_{x \to -3} \left(\frac{x^2 - 2x - 15}{(x+3)}\right) =$ $\quad \frac{9+6-15}{-3+3} = \frac{0}{0}$

so, factor the numerator

$$\lim_{x \to -3} \frac{(x-5)\cancel{(x+3)}}{\cancel{(x+3)}} = -3 - 5 = -8$$

5.) Let: $f(x) = \begin{cases} 4x^2 + 3x - 2, if \ldots . x < 1 \\ 5 + x, if \ldots\ldots\ldots\ldots . . x \geq 1 \end{cases}$

Find:

a. $\lim_{x \to 1-} f(x) =$ $4(1)^2 + 3(1) - 2 = 4 + 3 - 2 = 5$

b. $\lim_{x \to 1+} f(x) =$ $5 + 1 = 6$

c. Does $\lim_{x \to 1} f(x)$ exist? no, because the limits approaching 1 from the left and the right are different

6.) **Continuity:** Let: $f(x) = \begin{cases} 8 - x, if \ldots\ldots x \leq 4 \\ x^2 - 3x, if \ldots . x > 4 \end{cases}$

a. Is $f(x)$ continuous at 4? yes

b. Why or Why Not? $f(4) = 4$

$\lim_{x \to 4^-} f(x) = 8 - 4 = 4$

$\lim_{x \to 4^+} f(x) = 4^2 - 3(4) = 4$

the function exists at 4 and the limits from the right and left are 4 so the function is continuous

7.) **Composite Functions:** Given $f(x) = 4x^2 + 8x$ and $g(x) = 4x - 6$, find:

a. $(f \circ g)(3) =$ $f(g(3)) = 192$ $g(3) = 4(3) - 6 = 6$ $f(6) = 4(6)^2 + 8(6)$ $= 144 + 48 = 192$

b. $(g \circ f)(3) =$ $g(f(3)) = 234$ $f(3) = 4(3)^2 + 8(3) =$ $= 36 + 24 = 60$ $g(60) = 4(60) - 6$ $= 234$

c. $(g \circ g)(5) =$ $g(g(x)) = 50$ $g(5) = 4(5) - 6 = 14$ $g(14) = 4(14) - 6 = 50$

d. $(g \circ f)(x) =$ $g(f(x)) =$

$4(4x^2 + 8x) - 6$

$= 16x^2 + 32x - 6$

8.) If a ball is thrown into the air with a velocity of 58 m/s, its height in meters after t seconds is given by $h = 62t - .79t^2$. Find the average velocity over the time interval [1,3]. ______________

average velocity = $\dfrac{\text{change in position}}{\text{time elapsed}}$

$$V = \frac{h(3) - h(1)}{3 - 1}$$

$$V = \frac{(62(3) - .79(3)^2) - 62(1) - .79(1)^2}{2}$$

$$V = \frac{(186 - 7.11) - (62 - .79)}{2}$$

$$V = \frac{178.89 - 61.21}{2} = \frac{117.68}{2} = \boxed{58.84 \text{ m/s}}$$

9.) The point (0,2) lies on a curve $f(x) = 2 - 3x^3$. If Q is the point $(1, f(1))$, find the slope of the secant line **PQ.** ______________________________

Q is the point $(1, f(1))$

$$f(1) = 2 - 3(1)^3$$
$$= 2 - 3$$
$$= -1$$

so point Q is $(1, -1)$
point P is $(0, 2)$

slope of line PQ

$$\frac{2-(-1)}{0-1} = \frac{3}{-1} = -3$$