

Math 114 - Review - Chapter 6: Set Theory, Venn Diagrams, Multiplication Principle, Permutations, & Combinations.

Set Theory

You should know the following definitions:

- (1) Set Builder Notation – Describes the set in words and is detailed enough so the set can be properly built.
For example $J = \{x|x \text{ is a multiple of } 5 \text{ greater than } 5 \text{ but less than } 25\}$
- (2) Roster Notation – Lists the elements of the set much like a team roster. The Roster Notation for the set described in the definition of set builder would be $\{10,15,20\}$
- (3) Element – a member of a set.
- (4) Union - To find the union of two sets you join all the elements of the two sets together in one set.

If $A = \{1,2,3\}$ and $B = \{3,4,5\}$ then $A \cup B = \{1,2,3,4,5\}$
**Don't list common elements more than once.
- (5) Intersection – The intersection of two sets is the list of elements the two sets have in common.
If $A = \{1,2,3\}$ and $B = \{3,4,5\}$ then $A \cap B = \{3\}$
- (6) Complement of a Set – The complement of a set is defined as the set that consists of elements in the universe U that are not in the set you are taking the complement of. For example, if the universe $U = \{1,2,3,4,5,6\}$ and $A = \{1,2,3\}$ and $B = \{3,4,5\}$ then:

For more complex problems remember your order of operations:

$\overline{A \cap B} = \{6\}$ First find the complement of A and the complement of B , then take the intersection

$\overline{A \cap B} = \{1,2,4,5,6\}$ First find the intersection of set A and B , then take the complement of that set

- (7) Count the number of elements of a set ($c(A)$)

If $A = \{1,2,3,4\}$, then $c(A) =$ the number of elements in set $A = 4$

- (8) Subset and Proper Subset

Subset - $A \subseteq B$, if every element in A is also an element in B – note set A can equal set B

Proper Subset - $A \subset B$, if every element in A is also an element in B AND set B has at least one element set A does not have – in other words set A cannot equal set B .

- (9) The Empty Set or the Null Set

The Empty Set or the Null Set is the set with NO ELEMENTS and can be noted as $\{\}$ or ϕ

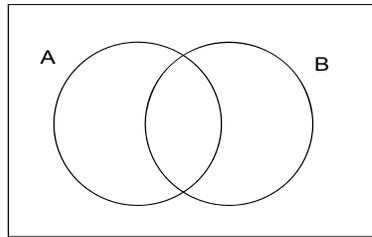
Now try problems 1-7 on the Sample Exam I

Venn Diagrams

1.) How to shade a Venn diagram to represent a given set:

What is a Venn Diagram?

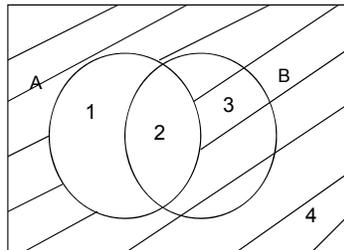
- A Venn Diagram can be used to illustrate relationships between sets
- Circles represent sets, the rectangular region represents the universal set, U. Both sets A and B are contained in the universal set, U.
- The Venn Diagram shown represents the relationship between sets A & B.



2

How can we represent \bar{A} ?

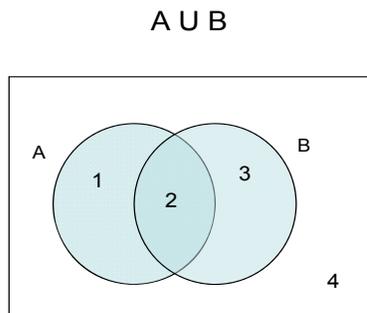
- $A = \{1,2\}$
- Now let's figure out what regions represent the complement of A. Since the complement of set A is defined as all the elements in the universe but NOT in A, the complement of A consists of regions in our rectangle but NOT in the circle representing set A.
 $\bar{A} = \{3,4\}$
- To represent the complement of set A we shade in regions {3,4}



5

How can we represent $A \cup B$?

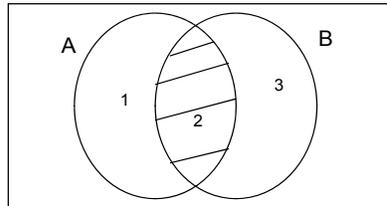
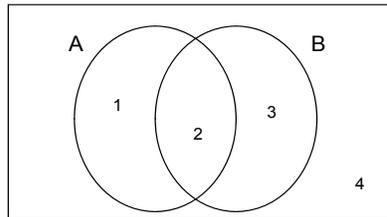
- $A = \{1,2\}$ and $B = \{2,3\}$
- Now let's figure out what regions represent $A \cup B$. Since the union of sets A and B is defined as the elements in either A or B or in both A and B, $A \cup B$ consists of regions {1,2,3}
- To represent the set $A \cup B$ we shade in regions {1,2,3}



4

How can we represent $A \cap B$?

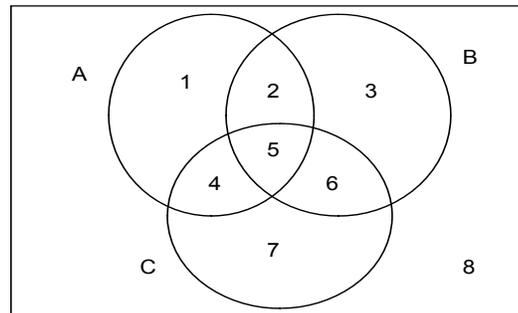
- First we want to number the regions in the Venn Diagram. For two sets there are four (4) regions.
- Set A consists of regions {1,2} and set B consists of regions {2,3}
- Anything in the universe but not in either set A or B is in region 4.
- Since $A \cap B$ is defined as the set consisting of elements in both A and B, then $A \cap B = \{2\}$. To represent this set we can shade region 2



3

A Venn Diagram & 3 Sets

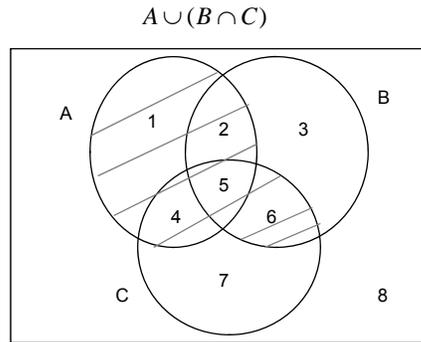
- A Venn Diagram can be used to represent the relationships between three sets
- The universal set, U, is represented by the rectangle that encompasses sets A, B, & C
- With 3 sets there are 8 regions
- Set A = {1,2,5,4}
- Set B = {2,3,5,6}
- Set C = {4,5,6,7}
- Region 8 consists of elements in the universe BUT NOT in sets A, B, or C
- The set $A \cap B \cap C$ are all the elements that are in sets A,B, & C and is represented by region {5}
- The set $A \cap C$, the overlap of circles A and C, is represented by regions {4,5}
- The set $A \cap B$, the overlap of circles A and B, is represented by regions {2,5}
- The set $B \cap C$, the overlap of circles B and C, is represented by regions {5,6}



7

How Can We Represent $A \cup (B \cap C)$?

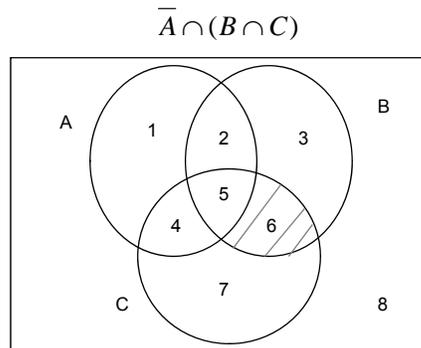
- We want to follow our order of operations, so we perform the operations in parenthesis first.
- $B \cap C = \{5,6\}$
- $A = \{1,2,4,5\}$
- So $A \cup (B \cap C)$ can be represented as $\{1,2,4,5\} \cup \{5,6\}$.
- Since union (U) means we list the elements in either set or in both sets, $\{1,2,4,5\} \cup \{5,6\} = \{1,2,4,5,6\}$
- We shade regions 1,2,4,5,6 to represent the set $A \cup (B \cap C)$.



8

How Can We Represent $\bar{A} \cap (B \cap C)$

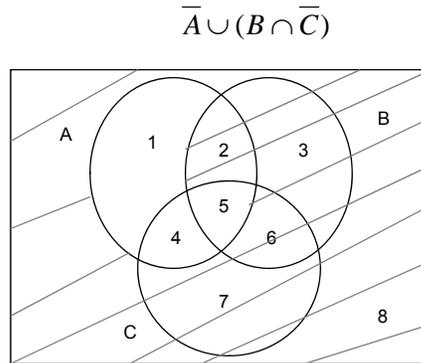
- We want to follow our order of operations, so we perform the operations in parenthesis first.
- $\bar{B} \cap C = \{5,6\}$
- \bar{A} is the set that includes all regions in the universe (our rectangle) BUT NOT in set A (circle A)
- $\bar{A} = \{3,6,7,8\}$
- We can now represent $\bar{A} \cap (B \cap C)$ as $\{3,6,7,8\} \cap \{5,6\}$
- To take the intersection of 2 sets we list the elements that are in BOTH sets.
- $\{3,6,7,8\} \cap \{5,6\} = \{6\}$



9

How Can We Represent $\overline{A} \cup (B \cap \overline{C})$

- First lets work in the parenthesis and determine what regions represent $B \cap \overline{C}$
- \overline{C} , the regions not in circle C, = {1,2,3,8}
- $B = \{2,3,5,6\}$
- $B \cap \overline{C} = \{2,3,5,6\} \cap \{1,2,3,8\} = \{2,3\}$
- $\overline{A} = \{3,6,7,8\}$
- So $\overline{A} \cup (B \cap \overline{C})$ can be represented as $\{3,6,7,8\} \cup \{2,3\} = \{2,3,6,7,8\}$
- We shade in regions 2,3,6,7,& 8



10

Now try problem 8 on the Sample Exam I

2.) How to use a Venn Diagram to solve a word problem:

Suppose that a group of 200 students are surveyed and ask which chatrooms they have joined. There are three chatrooms in our survey; one for skateboarding, one for bicycling, and one for college students.

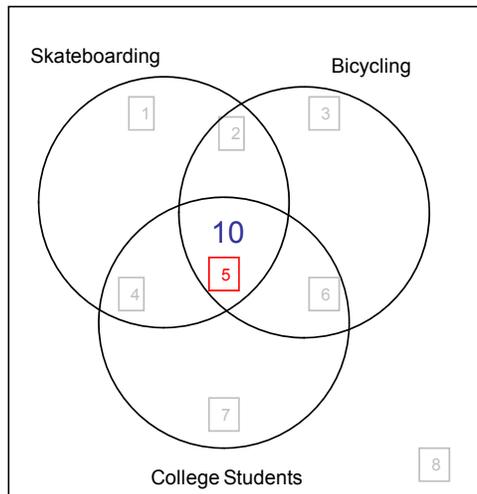
- 90 students joined the room for skateboarding;
- 50 students joined the room for bicycling;
- 70 students joined the room for college students;
- 15 students joined rooms for skateboarding and college students;
- 12 students joined rooms for bicycling and college students;
- 25 students joined rooms for skateboarding and bicycling;
- 10 students joined all three rooms.

- 1.) How many students joined the room for skateboarding OR bicycling?
- 2.) How many students did not join any of these three rooms?
- 3.) How many students joined the bicycling AND skateboarding rooms BUT NOT the room for college students?
- 4.) How many students joined EXACTLY 1 of these rooms?
- 5.) How many students joined AT MOST 2 of these rooms?

This problem seems too difficult to solve! But it isn't. You just need to use a Venn Diagram to represent the relationship between the three chat rooms and the answers to all 5 of these questions will be perfectly clear.

For this type of problem we fill in our regions for our Venn Diagram. We use the information given to fill in the number of students in each region. We start from the bottom and work our way to the top.

- 90 students joined the room for skateboarding;
- 50 students joined the room for bicycling;
- 70 students joined the room for college students;
- 15 students joined rooms for skateboarding and college students;
- 12 students joined rooms for bicycling and college students;
- 25 students joined rooms for skateboarding and bicycling;
- [10 students joined all three rooms.](#)

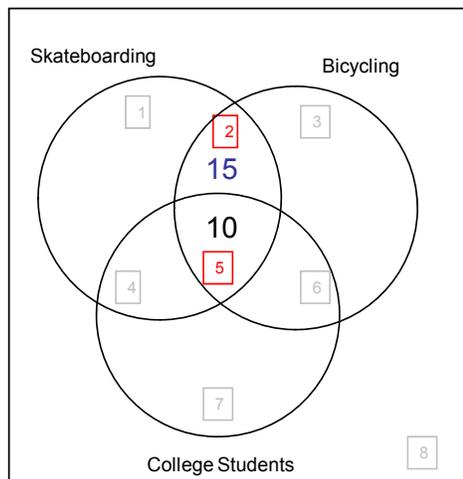


First we draw our Venn Diagram representing three sets, one set for Skateboarding, one set for Bicycling, and one set for College Student chat rooms. Then we label our regions and start putting in the number of elements or students for each region. 10 students joined all three rooms means these 10 students are in all three circles - the intersection of circles for skateboarding, bicycling, and college students. This is represented by **region 5** - we need to put in a **10** in **region 5**.

Our next piece of information is:

[25 students joined rooms for skateboarding and bicycling](#)

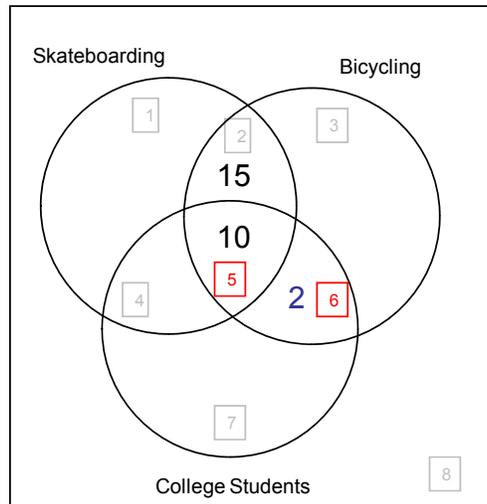
What do these 25 students represent? They are the the set of students who joined the Skateboarding chatroom **AND** the Bicycling chatroom or $n(\text{Skateboarding} \cap \text{Bicycling})$. These 25 students are in both the Skateboarding circle AND the Bicycling circle. This intersection is represented by **regions 2 + 5** - we already have 10 students in region 5 so the number of students to put in **region 2** = $25 - 10$ or **15**.



Our next piece of information is:

12 students joined rooms for bicycling and college students

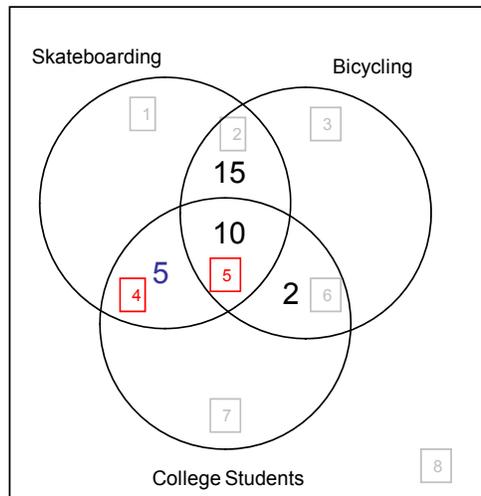
What do these 12 students represent? They are the the students who joined the Bicycling chatroom **AND** the College Students chatroom or $n(\text{Bicycling} \cap \text{College Students})$. These 12 students are in both the Bicycling circle AND the College Student circle. This intersection is represented by **regions 5 + 6** - we already have 10 students in region 5 so the number of students to put in **region 6** = $12 - 10$ or **2**.



Our next piece of information is:

15 students joined rooms for skateboarding and college students

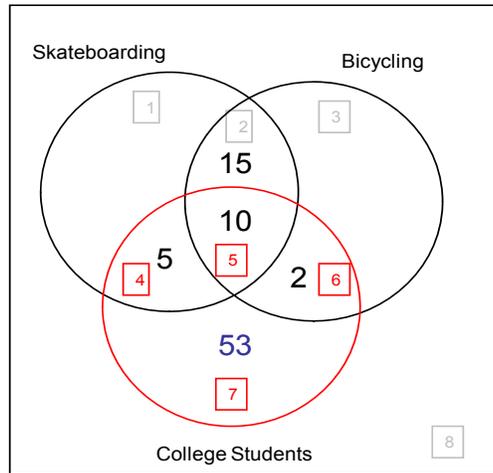
What do these 15 students represent? They represent the students who joined the Skateboarding chatroom **AND** the students who joined the College Students chatroom or $n(\text{Skateboarding} \cap \text{College Students})$. These 15 students are in both the Skateboarding circle AND the College Student circle. This intersection is represented by **regions 4 + 5** - we already have 10 students in region 5 so the number of students to put in **region 4** = $15 - 10$ or **5**.



Our next piece of information is:

70 students joined the room for college students

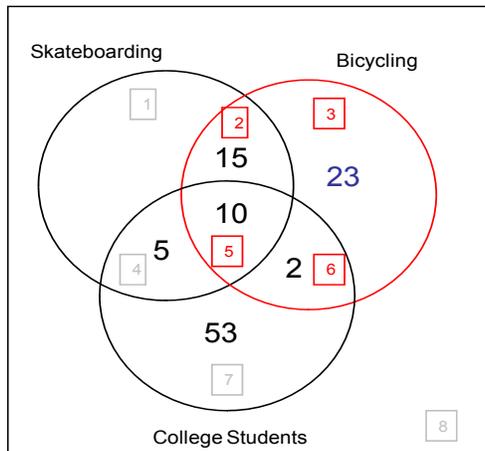
What do these 70 students represent? They represent ALL the students who joined the College Students chat room or $n(\text{College Students})$. This is represented by **regions 4 + 5 + 6 + 7** - the sum of the students in these four regions = 70. We already have 5 students in region 4, 10 students in region 5, and 2 students in region 6 so the number of students to put in **region 7** = $70 - (10 + 5 + 2) = 53$.



Our next piece of information is:

50 students joined the room for bicycling

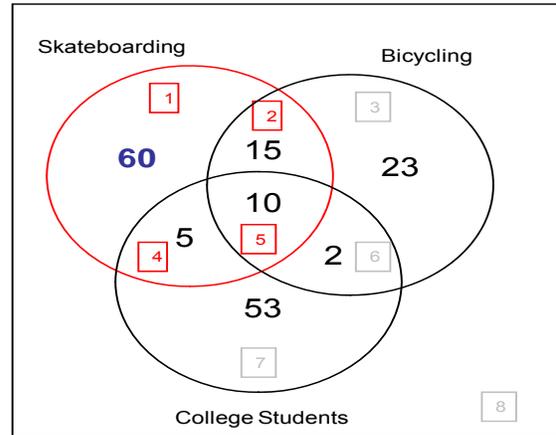
What do these 50 students represent? They represent ALL the students who joined the Bicycling chat room or $n(\text{Bicycling})$. This is represented by **regions 2 + 3 + 5 + 6** - the sum of the students in these four regions = 50. We already have 15 students in region 2, 10 students in region 5, and 2 students in region 6 so the number of students to put in **region 3** = $50 - (15 + 10 + 2) = 23$.



Our next piece of information is:

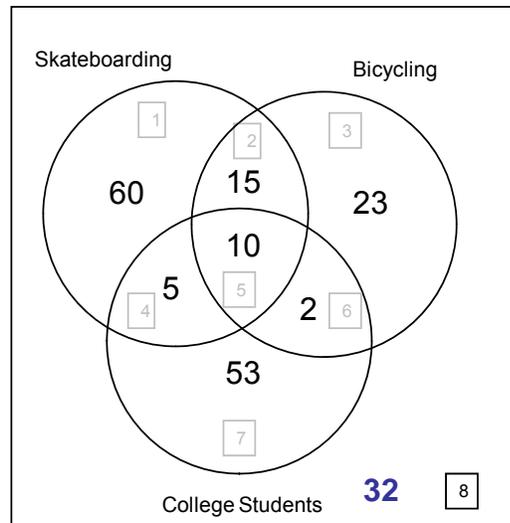
90 students joined the room for skateboarding

What do these 90 students represent? They represent ALL the students who joined the Skateboarding chatroom or $n(\text{Skateboarding})$. This is represented by regions 1 + 2 + 4 + 5 - the sum of the students in these four regions = 90. We already have 15 students in region 2, 5 students in region 4, and 10 students in region 5 so the number of students to put in region 1 = $90 - (15 + 5 + 10) = 60$.



We have now filled in regions 1 thru 7, we only have to fill in region 8. What students does region 8 represent? Region 8 represents the students surveyed who did not join any of the three chatrooms in our survey. These students are not in any of the circles that represent our sets. The sum of ALL 8 regions must add up to all the students we surveyed - our universe - or 200 students. Therefore, the number of students in region 8 = $200 - (60 + 15 + 23 + 5 + 10 + 2 + 53) = 32$.

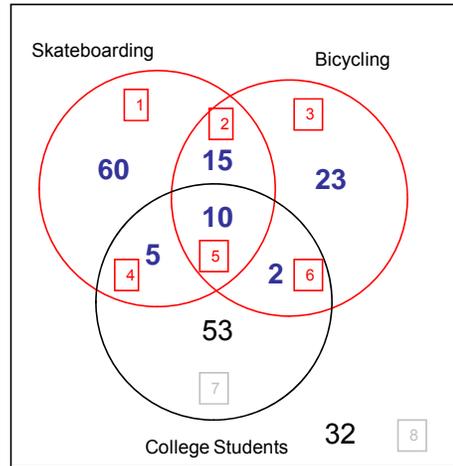
Now that we have all our regions filled in we can answer our five questions.



Question 1:

How many students joined the room for skateboarding OR bicycling?

This is asking us for the union of the skateboarding and bicycling chatrooms or $n(\text{Skateboarding} \cup \text{Bicycling})$. For union we bring all the members of the skateboarding and bicycling chatrooms together. We want all the regions in the skateboarding and bicycling circle (remember don't count regions more than once). So the students we are interested in are in regions 1 + 2 + 3 + 4 + 5 + 6 or

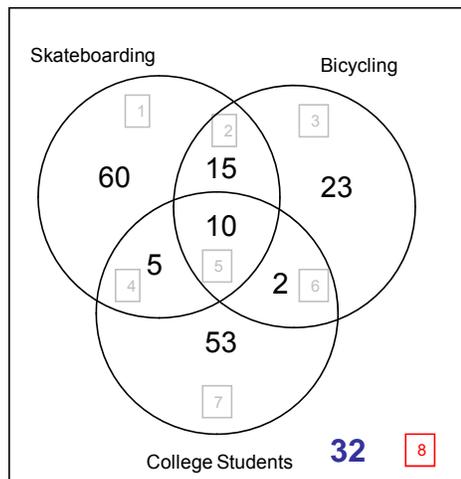


$$60 + 15 + 23 + 5 + 10 + 2 = 115 \text{ students}$$

Question 2:

How many students did not join any of these three rooms?

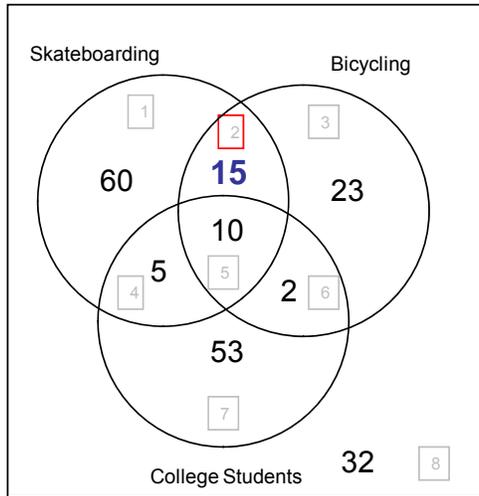
The students who did not join any groups are not in ANY of the three circles. These students are in region 8. So the number of students who did not join any of these three chatrooms = 32 students



Question 3:

How many students joined the bicycling AND skateboarding rooms BUT NOT the room for college students?

We want the students who joined bicycling AND skateboarding - this represents the intersection of the skateboarding and bicycling sets - regions 2 + 5. The next part of the question tells us that we do not want students in the College Students set, so we do not want region 5 since this region while it is in the intersection of skateboarding and bicycling it is also in the college students circle, we only want region 2. The answer is 15 students.



Question 4:

How many students joined EXACTLY 1 of these rooms?

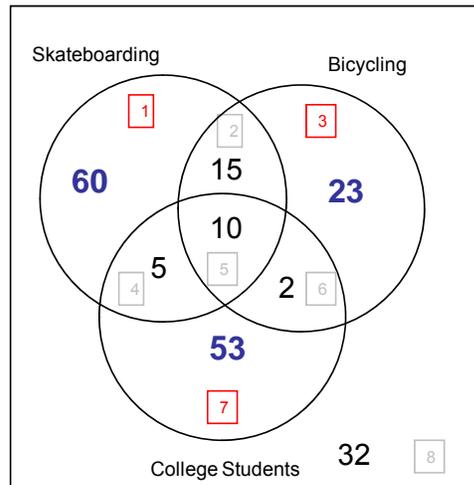
The students who joined EXACTLY 1 of the rooms will be in regions that are only in 1 circle. These regions include 1, 3, and 7, we will add all the students in these regions to get our answer.

$$60 + 23 + 53 = 136 \text{ students}$$

Note:

Students who joined exactly 2 chatrooms will be in regions in exactly 2 circles, these regions are 2,4, and 6.

Students who joined exactly 3 chatrooms will be in regions in exactly 3 circles, this region is 5.



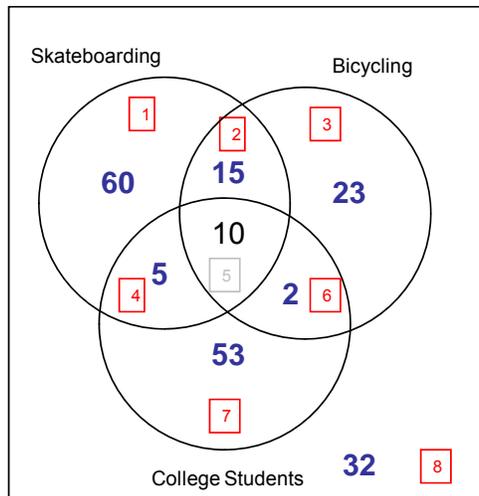
Question 5:

How many students joined AT MOST 2 of these rooms?

AT MOST 2 means the number of rooms we want a student to join is LESS THAN OR EQUAL TO 2. (≤ 2 rooms). We want regions that are in 2 circles, 1 circle, or none of the circles. These include regions 1,2,3,4,6,7,and 8. We add up all these regions to get our answer.

$$60+15+23+5+2+53+32 = 190 \text{ students}$$

NOTE: If the question had asked for the number of students who joined AT LEAST 2 of these rooms we would be interested in regions that are in 2 OR MORE circles (at least mean means ≥ 2). We would want to add together the students in regions 2,4,5,and 6.



Now try problems 9 through 12 on the Sample Exam I

Counting Techniques

The Multiplication Principle – If a task consists of a sequence of choices in which there are p selections for the first choice, q selections for the second choice, r selections for the third choice, and so on, then the task of making these selections can be done in $p \cdot q \cdot r \dots$ different ways.

For example: How many different outfits can be put together if I have 3 pair of pants, 7 shirts, two pairs of shoes, and 4 sweaters?

$$\text{Number of outfits} = 3 \cdot 7 \cdot 2 \cdot 4 = 168$$

Sample Multiplication Principle Problems:

- (1) How many different cars can be made if there are 6 different car models, 2 interior colors, 5 exterior colors, and 2 stereo options?

| | | | | | | | | |
|-------------------|-----------------|---|----------------------|---|----------------------|---|---------------------|------|
| Number of choices | 6 | X | 2 | X | 5 | X | 2 | =120 |
| | # of Car Models | | # of interior colors | | # of exterior colors | | # of stereo options | |

- (2) How many different sets of answers are possible on a test with 7 true-false questions?

| | | | | | | | | | | | | | | |
|--------------|----|---|----|---|----|---|----|---|----|---|-------------|--|----|--|
| # of choices | 2 | X | 2 | X | 2 | X | 2 | X | 2 | = | $2^7 = 128$ | | | |
| | Q1 | | Q2 | | Q3 | | Q4 | | Q5 | | Q6 | | Q7 | |

(3) How many two- or three-letter initials for people are possible if no letter can be repeated?

$$\begin{array}{rcccl} \text{Two Letter Initials} & & \text{OR} & & \text{Three Letter Initials} \\ (26 * 26) & & + & & (26*26*26) & = \\ 676 & + & 17,576 & = & 18,252 \end{array}$$

Now try problems 13, 14, 15, 16, 17, & 24 on the Sample Exam I

Permutations: ORDER MATTERS!

The number of arrangements of n objects using r ≤ n of them, in which

1. A permutation is an arrangement or sequence of selections of elements from a single set.
2. Once an object is selected it can't be used again – i.e. repetitions are not allowed
3. Order MATTERS

The permutation formula is:

$$P(n, r) = \frac{n!}{(n - r)!}$$

Example: From a group of ten fifth graders, 6 are selected and arranged in a line, how many different arrangements are there? Order matters here – if you are first in line it is different than being last in line.

$$P(10, 6) = \frac{10!}{(10 - 6)!} = 151,200$$

Another type of permutation problem involves calculating the number of arrangements of letters that can be made from a given word. This is a unique problem because the letters in the word may not be distinct. To solve this problem use the following formula:

The number of permutations of n objects, of which n₁ are of one kind, n₂ are of a second kind,, and n_k are of a kth kind is given by:

$$\frac{n!}{n_1! \times n_2! \times n_3! \times \dots \times n_k!}$$

where: n₁ + n₂ + n₃ + n_k = n

Example: How many distinct “words” can be formed from the word STEELERS ? Note there are 2 s, 1 t, 3 e, 1 l, 1 r in the word Steelers

$$\# \text{ of Words} = \frac{8!}{2! \times 1! \times 3! \times 1! \times 1!} = 3,360$$

Now how many words can you form from the word Champion or Losers?

Combinations: ORDER DOESN'T MATTER!

The number of arrangements of n objects using $r \leq n$ of them, in which

1. A combination selects elements from a single set
2. Once an object is selected it can't be used again – i.e. repetitions are not allowed
3. Order DOES NOT MATTER!

The combination formula is:

$$C(n, r) = \frac{n!}{(n-r)!r!}$$

Example 1: From a group of ten fifth graders 6 are selected for a committee, how many different arrangements are there? Note: In a committee order does not matter.

$$C(10, 6) = \frac{10!}{(10-6)!6!} = 210$$

Example 2: An urn contains fifteen balls - 5 red balls, 7 blue balls, and 3 yellow balls. If you pick a sample of five balls from the urn in how many ways can you pick

a.) The sample of 5?

$$C(15, 5) = 3003$$

b.) five red balls

$$C(5, 5) * C(10, 0) = 1$$

c.) three red balls and two yellow balls

$$C(5, 3) * C(3, 2) * C(7, 0) = 30$$

d.) at least four red balls

$$C(5, 4) * C(10, 1) + C(5, 5) * C(10, 0) = 50$$

You should know when to use the permutations formula (order matters) and when to use the combinations formula (order doesn't matter).

Now try problem 18 – 27 on the Sample Exam I