

Math 113 – Sample Exam: Systems of Linear Equations

Solve each system of linear equations using **substitution**

$$\begin{aligned}
 1.) \quad & 3x - y = 1 \\
 & x + 2y = -9 \\
 & \quad -2y \quad -2y \\
 & x = -9 - 2y \\
 & 3(-9 - 2y) - y = 1 \\
 & -27 - 6y - y = 1 \\
 & -27 - 7y = 1 \\
 & +27 \quad +27 \\
 & -7y = 28 \\
 & \quad -7 \quad -7 \\
 & y = -4
 \end{aligned}$$

$$\begin{aligned}
 2.) \quad & x - y = 5 \\
 & -3x + 2y = 24
 \end{aligned}$$

$$\begin{aligned}
 & x - y = 5 \\
 & \quad +y \quad +y \\
 & x = 5 + y \\
 & -3(5 + y) + 2y = 24 \\
 & -15 - 3y + 2y = 24 \\
 & -15 - y = 24 \\
 & +15 \quad +15 \\
 & -y = 39 \\
 & \quad y = -39
 \end{aligned}$$

$$\begin{aligned}
 3.) \quad & r + s = 0 \\
 & r - s = 5 \\
 & \quad +s \quad +s \\
 & r = 5 + s \\
 & (5 + s) + s = 0 \\
 & 5 + s + s = 0 \\
 & 5 + 2s = 0 \\
 & * -5 \quad -5 \\
 & \frac{2s}{2} = \frac{-5}{2} \\
 & s = -\frac{5}{2}
 \end{aligned}$$

$$\begin{aligned}
 & 3x - y = 1 \\
 & 3x - (-4) = 1 \\
 & 3x + 4 = 1 \\
 & \quad -4 \quad -4 \\
 & \frac{3x}{3} = \frac{-3}{3} \\
 & x = -1
 \end{aligned}$$

$$\begin{aligned}
 & x - y = 5 \\
 & x - (-39) = 5 \\
 & x + 39 = 5 \\
 & \quad -39 \quad -39 \\
 & x = -34
 \end{aligned}$$

$$\begin{aligned}
 & r + s = 0 \\
 & r - \frac{s}{2} = 0 \\
 & \quad +\frac{s}{2} \quad +\frac{s}{2} \\
 & r = \frac{s}{2}
 \end{aligned}$$

check answer

$$\begin{aligned}
 & 3(-1) - (-4) = 1 \\
 & -3 + 4 = 1 \checkmark \\
 & -1 + 2(-4) = -9 \\
 & -1 - 8 = -9 \checkmark
 \end{aligned}$$

$(-1, -4)$

check answer

$$\begin{aligned}
 & -34 - (-39) = 5 \\
 & -34 + 39 = 5 \checkmark \\
 & -3(-34) + 2(-39) = 24 \\
 & 102 - 78 = 24 \checkmark
 \end{aligned}$$

$(-34, -39)$

check answer

$$\begin{aligned}
 & \frac{5}{2} - \frac{5}{2} = 0 \checkmark \\
 & \frac{5}{2} - \left(-\frac{5}{2}\right) = 5 \\
 & \frac{10}{2} = 5 \checkmark
 \end{aligned}$$

$\left(\frac{5}{2}, -\frac{5}{2}\right)$

$$4.) \begin{aligned} 3x - 5y &= 16 \\ 15x + 5y &= 20 \\ -15x & \qquad -15x \end{aligned}$$

$$\frac{5y}{5} = \frac{20}{5} - \frac{15x}{5}$$

$$y = 4 - 3x$$

$$3x - 5(4 - 3x) = 16$$

$$3x - 20 + 15x = 16$$

$$18x - 20 = 16$$

$$+20 \quad +20$$

$$\frac{18x}{18} = \frac{36}{18}$$

$$x = 2$$

$$15(2) + 5y = 20$$

$$30 + 5y = 20$$

$$-30 \quad -30$$

$$\frac{5y}{5} = \frac{-10}{5}$$

$$y = -2$$

check answer

$$3(2) - 5(-2) = 16$$

$$6 + 10 = 16 \checkmark$$

$$15(2) + 5(-2) = 20$$

$$30 - 10 = 20 \checkmark$$

$$\underline{(2, -2)}$$

Solve each system of linear equations using **elimination**

$$5.) \begin{aligned} 5x + y &= 4 \\ 3x + 2y &= 1 \end{aligned}$$

$$3R_1 \rightarrow 3(5x + y = 4)$$

$$-5R_2 \rightarrow -5(3x + 2y = 1)$$

$$\begin{aligned} 15x + 3y &= 12 \\ -15x - 10y &= -5 \end{aligned}$$

$$-7y = 7$$

$$\frac{-7y}{-7} = \frac{7}{-7}$$

$$y = -1$$

$$5x + (-1) = 4$$

$$\frac{5x}{5} + 1 = \frac{5}{5}$$

$$x = 1$$

check answer

$$5(1) + (-1) = 4$$

$$5 - 1 = 4 \checkmark$$

$$3(1) + 2(-1) = 1$$

$$3 - 2 = 1 \checkmark$$

$$\underline{(1, -1)}$$

$$6.) \begin{aligned} 2x - 2y &= 12 \\ -2x + 3y &= 10 \end{aligned}$$

$$y = 22$$

$$2x - 2(22) = 12$$

$$2x - 44 = 12$$

$$+44 \quad +44$$

$$\frac{2x}{2} = \frac{56}{2}$$

$$x = 28$$

check answer

$$2(28) - 2(22) = 12$$

$$56 - 44 = 12 \checkmark$$

$$-2(28) + 3(22) = 10$$

$$-56 + 66 = 10 \checkmark$$

$$\underline{(28, 22)}$$

$$7.) \begin{cases} 2x - y = 1 \\ 3x - y = 6 \end{cases}$$

$$\begin{aligned} -3R_1 &\rightarrow -3(2x - y = 1) \\ 2R_2 &\rightarrow 2(3x - y = 6) \end{aligned}$$

$$\begin{array}{r} -6x + 3y = -3 \\ 6x - 2y = 12 \\ \hline y = 9 \end{array}$$

$$\begin{aligned} 2x - 9 &= 1 \\ +9 &+9 \\ \hline 2x &= 10 \\ \frac{2x}{2} &= \frac{10}{2} \\ x &= 5 \end{aligned}$$

check answer

$$\begin{aligned} 2(5) - 9 &= 1 \\ 10 - 9 &= 1 \checkmark \\ 3(5) - 9 &= 6 \\ 15 - 9 &= 6 \checkmark \end{aligned}$$

(5, 9)

$$8.) \begin{cases} 2x - 3y = -1 \\ 10x + y = 11 \end{cases}$$

$$\begin{aligned} -5R_1 &\rightarrow -5(2x - 3y = -1) \\ R_2 & \\ \hline -10x + 15y &= 5 \\ 10x + y &= 11 \\ \hline 16y &= 16 \\ \frac{16y}{16} &= \frac{16}{16} \\ y &= 1 \end{aligned}$$

check answer

$$\begin{aligned} 2(1) - 3(1) &= -1 \\ 2 - 3 &= -1 \checkmark \\ 10(1) + 1 &= 11 \\ 10 + 1 &= 11 \checkmark \end{aligned}$$

$$\begin{aligned} 2x - 3(1) &= -1 \\ 2x - 3 &= -1 \\ +3 &+3 \\ \hline 2x &= 2 \\ \frac{2x}{2} &= \frac{2}{2} \\ x &= 1 \end{aligned}$$

(1, 1)

Determine if the following systems of linear equations have (a.) one solution. (b.) infinitely many solutions, or (c.) no solutions. Find all the solutions, if possible. Then give a geometric interpretation of the linear system. (Lines intersect, are parallel, or it is the same line)

$$9.) \begin{cases} -6x + 4y = 7 \\ 3x - 2y = 4 \end{cases}$$

$$\begin{array}{l} R_1 \\ 2R_2 \end{array} \begin{array}{l} -6x + 4y = 7 \\ 6x - 4y = 8 \end{array}$$

$$0 = 15$$



not true

so, there is no solution

no solution

These are parallel lines.

$$10.) \begin{cases} 3x - 4y = 1 \\ 2x + 3y = 12 \end{cases}$$

$$\begin{array}{l} 2R_1 \\ -3R_2 \end{array} \begin{array}{l} 6x - 8y = 2 \\ -6x - 9y = -36 \end{array}$$

$$\frac{-17y}{-17} = \frac{-34}{-17}$$

$$y = 2$$

$$3x - 4(2) = 1$$

$$\begin{array}{r} 3x - 8 = 1 \\ \quad +8 \quad +8 \end{array}$$

$$\frac{3}{3}x = \frac{9}{3}$$

$$x = 3$$

(3, 2)

The two lines intersect.

$$11.) \begin{aligned} -4x + y &= 2 \\ 8x - 2y &= -4 \end{aligned}$$

$$2R_1 \rightarrow \begin{aligned} -8x + 2y &= 4 \\ 8x - 2y &= -4 \end{aligned}$$

$0 = 0 \leftarrow$ infinitely many solutions

$$\begin{aligned} -4x + y &= 2 \\ -y &= -y \\ \hline -4x &= -y + 2 \\ \frac{-4}{-4}x &= \frac{-y}{-4} + \frac{2}{-4} \end{aligned}$$

$$x = \frac{1}{4}y - \frac{1}{2}$$

$$\left(\frac{1}{4}y - \frac{1}{2}, y \right)$$

These equations are for
the same line.

- 12.) The Blue & Gold movie theater charges \$9.00 for adults and \$7.00 for senior citizens. On a day when 325 people paid an admission, the total receipts were \$2495. How many who paid were adults? How many were seniors?

$$\begin{aligned} x &= \# \text{ of adults} \\ y &= \# \text{ of seniors} \end{aligned}$$

$$\begin{aligned} 9(110) + 7(215) &= 2495 \\ 990 + 1505 &= 2495 \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{1} \quad x + y &= 325 \\ \textcircled{2} \quad 9x + 7y &= 2495 \end{aligned}$$

$$\begin{aligned} -9R_1 \rightarrow \begin{aligned} -9x - 9y &= -2925 \\ 9x + 7y &= 2495 \\ \hline -2y &= -430 \\ \frac{-2}{-2} &= \frac{-430}{-2} \\ y &= 215 \end{aligned} \end{aligned}$$

Adults 110

Seniors 215

$$\begin{aligned} x + y &= 325 \\ x + 215 &= 325 \\ -215 &= -215 \end{aligned}$$

$$x = 110$$

13.) A child has 25 coins in a piggy bank consisting of dimes and quarters. The total value of the coins is \$3.70. Find the number of dimes and the number of quarters.

$x = \#$ of dimes
 $y = \#$ of quarters

① $x + y = 25$
 ② $.1x + .25y = 3.70$

$.1(17) + .25(8) = 3.70$
 $1.70 + 2.00 = 3.70 \checkmark$

$-0.1 R_1 \rightarrow -0.1x - 0.1y = -2.5$
 $.1x + .25y = 3.70$

$\frac{.15y = 1.2}{.15} \quad \frac{1.2}{.15}$

$y = 8$

$x + 8 = 25$
 $-8 \quad -8$
 $x = 17$

Dimes 17
 Quarters 8

14.) A restaurant manager wants to purchase 200 sets of dishes. One design costs \$25 per set, while another costs \$45 per set. If she only has \$7400 to spend, how many of each design should be ordered?

$x = \#$ of \$25 sets of dishes
 $y = \#$ of \$45 sets of dishes

① $x + y = 200$
 ② $25x + 45y = 7400$

$25(80) + 45(120) = 7400$
 $2000 + 5400 = 7400 \checkmark$

$-25 R_1 \rightarrow -25x - 25y = -5000$
 $25x + 45y = 7400$

$\frac{20y = 2400}{20} \quad \frac{2400}{20}$

$y = 120$

\$25 set 80 sets

\$45 set 120 sets

$x + 120 = 200$
 $-120 \quad -120$
 $x = 80$