<u>Chapter 5 – Finance</u> – The first part of this review will explain the different interest and investment equations you learned in section 5.1 through 5.4 of your textbook and go through several examples. The second part of this review will give you various sample problems to work on – you should know how to do all these sample problems for Exam II. The sample exam is posted to the Math 113 webpage – please come to the Math Resource Center if you need help!

The sum of money you deposit into a savings account or borrow from a bank is called the principal. The fee to borrow money is called interest. When you borrow money you pay back the principal and interest to your lender. When you deposit money into a savings account or other investment, the bank pays you back the principal and the interest earned. Interest is calculated as a percent of the money borrowed, a percent of the principal. Depending on the type of loan, interest can be calculated in a variety of ways.

<u>Simple Interest Loans (Section 5.1) :</u>

Simple Interest involves a single payment and the interest computed on the principal only. The equation for simple interest is a linear function. If the problem refers to a simple interest rate, then you know you need to use simple interest rate formulas.

Equation #1:

$I = \Pr t$

where :

I = the amount of interest paid for borrowing the money
P= the principal or the amount of money you borrowed from the bank
r = is the simple interest rate – this is a per annum rate (i.e. yearly)
t = the amount of time the money is borrowed for – this needs to be in years if the problem gives you *t* in months then you need to divide the number of months by 12 to convert *t* into years.

Equation #2:

A = (P+Prt)=P*(1+rt)

where:
A = the future value - the total amount the borrower owes at the end of the loan period – this is Principal plus Interest.
I = the amount of interest paid for borrowing the money
P= the principal or the amount of money you borrowed from the bank
r = is the simple interest rate – this is a per annum rate (i.e. yearly)
t = the amount of time the money is borrowed for – this needs to be in years if the problem gives you *t* in months then you need to divide the number of months by 12 to convert *t* into years.

Equation #3:

I = A – P

where:
A = the total amount you owe at the end of the loan period – this is Principal plus Interest.
I = the amount of interest paid for borrowing the money

P= the principal or the amount of money you borrowed from the bank

Problems with simple interest – in these problems you need to identify what you are given and what you are solving for:

1. Find the interest due on each loan if $100 is borrowed for 6 months at an 8% **simple interest rate.**

For this problem you need to solve for the interest owed on the loan or I. Use Equation #1:

I = Prt

P = Principal or the amount of the loan = $100
r = the simple interest rate = 8% per year
t= time of the loan BUT this must be in years so, t = (8/12)

I = Prt = (100)*(.08)*(8/12) = $5.33

2. Find the **simple interest rate** for a loan where $500 is borrowed and the amount owed after 8 months is $600.

This is a simple interest problem where you are solving for the simple interest rate or r. You can use Equation #1 and solve for r.

If I = Prt, then $r = \frac{I}{Pt}$

You are given t = amount of time the money is borrowed = 8 months BUT you need to convert this to years by dividing by 12 so, t = (8/12). Then you need to calculate I. You are given P (the principal, i.e. the amount borrowed) this equals $500 and you are also given A – the amount owed at the end of the loan. A = $600. You can use equation #3 to calculate I.

I = A-P = 600 – 500 = 100

then,

$$r = \frac{I}{Pt} = \frac{100}{500*(\frac{8}{12})} = .30 = 30\%$$

3. What is the term of a loan where $600 was borrowed at a **simple interest** of 8% and the interest paid on the loan was $156.

You know you need to use simple interest formulas and in this problem you need to solve for t the term of the loan or the amount of time you borrow the money for. You can use Equation #1 and solve for t.

If I = Prt, then $t = \frac{I}{\text{Pr}}$, t will be in years.

This problem is pretty straight forward, you are given I = interest paid = $156, P= amount borrowed = $600, and r = simple interest rate = 8%, so

$$t = \frac{I}{\Pr} = \frac{156}{600 * .08} = 3.25 years.$$

<u>Discounted Loans</u>:

A Discounted Loan is a loan where the lender deducts the interest due on the loan from the amount of money borrowed prior to giving the money to you. For instance, you may want to borrow $1000 from a bank if the loan is discounted the bank does not give you $1000, they give you $1000 minus the interest due, for example say the interest on this loan was $100 – you would receive $900 from the bank. However you owe the bank $1000 at the end of the loan period. Key words to identifying a loan as a discounted loan are: discounted loan and proceeds.

Equation #4:

R=L-Lrt = L*(1-rt)

where:

R= the amount of money the bank gives you or the **proceeds**
L = the amount you need to pay back to the bank
r = the discounted interest rate, this is a yearly or per annum rate
t = the amount of time you have to pay the back the bank – this is in **years**
Lrt = the discount or the amount deducted from the loan – the interest on the loan.

You can solve Equation #4 for L, for t or for r.

Problems with discounted loans – again in these problems you need to identify what you are given and what you are solving for:

1. A borrower signs a note for a discounted loan and agrees to pay the lender $1000 in 9 months at a 10% rate of interest. How much does the borrower receive?

For this problem you need to use equation #4 to solve for R – the proceeds or the amount of money the borrower receives. You are given L = the amount the borrower owes the bank = $1000, r = 10%, and t = 9 months = (9/12) years, so:

R = L*(1-rt) = $1000*(1-(.10)*(9/12)) = $925

2. You wish to borrow $10,000 for 3 months. If the person you are borrowing from offers a **discounted loan** at 8%, how much must you repay at the end of the loan?

For this problem you use equation #4 but now you are solving for L, the amount you need to pay back to the bank. You are given R – the proceeds – or the amount you receive from the bank = $10,000, r = the interest rate = 8%, and t = the time you have to pay back the loan = 3 months – you need to convert this to years = (3/12) years, so:

If R = L*(1-rt), then:

$$L = \frac{R}{(1-rt)} = \frac{10000}{(1-(.08)*(\frac{3}{12}))} = \$10{,}204.08$$

3. How much must you repay the bank if the **proceeds** of your loan are $1500 for 18 months at 10%?

You need to solve for L= the loan amount or the amount or money you need to repay the bank. You are given the value of R – the proceeds = the amount of money you get from the bank = $1500. You need to use equation #4 and solve for L.

$$L = \frac{R}{(1-rt)} = \frac{1500}{(1-(.10)*(\frac{18}{12}))} = \$1,764.71$$

<u>Compounded Interest (Section 5.2)</u>

If at the end of a payment period the interest is reinvested at the same rate, then over the next payment period you will earn interest on the interest from the first payment period plus the original principal. Interest paid on interest reinvested is called compound interest. The formula for compound interest is an exponential function, so ***if the rates and lengths of the loan are the same a compounded investment will earn more money than a simple interest investment.***

Compounded interest is with loans – here you owe money so you look for a loan with lower interest rate and fewer compound periods per year. Compound interest is also used with savings accounts and CDs – here you are earning money on the interest from the last payment period – so you want a higher interest rate and multiple compound periods.

The following payment periods (number times interest is compounded per year) apply to compounded interest problems:

Annually	Once per Year (m = 1)
Semiannually	Twice per year (m = 2)
Quarterly	4 times per year (m = 4)
Monthly	12 times per year (m = 12)
Daily	365 times per year (m = 365)

Equation #5 – Compound Interest Formula:

$A = P(1+i)^n$

where:

A = the total amount owed (interest plus principal) or the total amount earned on a savings account or CD (principal plus interest) after n payment periods.
P= the principal or original amount of the loan or investment
i = $\frac{r}{m}$ the interest rate for the compound period is calculated by dividing the annual interest rate by the number of compound periods in a year.
n = the number of deposits made for the duration of the annuity (m * t)
m= the number of compound periods in a year
t = length of the annuity in years

Compound interest problems using this formula involve a single payment and the amount of interest earned over the length of the investment/loan.

Problems with Compounded Interest:

1. If $5000 is invested at an annual rate of interest of 10%, what is the amount after 5 years if the compounding takes place (a) annually (b) monthly or (c) daily?

For this problem you need to use the Compounded Interest formula and solve for A_n. The value of the principal – P remains the same for each problem but the value of i and n will change depending on the number of compound periods that take place – If compounded annually i = annual rate/1 and n = 1 * number of years, if compounded monthly i = annual rate/12 and n = 12 * 5, and if compounded daily i = annual rate/365 and n = 365*5.

a. $A = P(1+i)^n = 5000\left(1+\left(\frac{.10}{1}\right)\right)^{(5\times 1)} = \$8{,}052.55$

b. $A = P(1+i)^n = 5000\left(1+\left(\frac{.10}{12}\right)\right)^{(5\times 12)} = \$8{,}226.54$

c. $A = P(1+i)^n = 5000\left(1+\left(\frac{.10}{365}\right)\right)^{(5\times 365)} = \$8{,}243.04$

2. You need $5000 and you need to take out a loan. Which loan would be the best loan for you to take? The time period for all the loans is 1 year.
   a. A discounted loan with an interest rate of 10.5%
   b. A loan with a rate of 10% compounded monthly
   c. A loan with a rate of 9.5% compounded daily

To solve this problem you need to compute how much money you would need to payback to the bank with each type of loan.

a. This is a discounted loan and you need to use the discounted loan equation and solve for L – the amount of money you need to payback to the bank. You need $5000 so this is the amount of money you want from the bank or your proceeds R = $5000, r = .105, and t =1

$$L = \frac{R}{(1-rt)} = \frac{5000}{(1-.105*1)} = \$5,586.59$$

b. This is a compounded interest loan and you need to use the compounded interest formula and solve for A_n – the amount you owe the bank Since the interest is compounded monthly for a period of one year i = (.10/12) and n = 12.

$$A = P(1+i)^n = 5000\left(1+\left(\frac{.10}{12}\right)\right)^{(1\times 12)} = \$5,523.57$$

c. This is a compounded interest loan and you need to use the compounded interest formula and solve for A_n – the amount you owe the bank. Since the interest is compounded daily for a period of one year i = (.095/365) and n = 365.

$$A = P(1+i)^n = 5000\left(1+\left(\frac{.095}{365}\right)\right)^{(1\times 365)} = \$5,498.23$$

Since the loan that is compounded daily gives us the lowest value for – the amount of money you owe the bank (principal plus interest), the best loan for you is loan c.

For more examples of problems with compounded interest you should review section 5.2 in your textbook and be able to do the homework problems in this section.

Annuities & Sinking Funds (Section 5.3)

An **Annuity** is a sequence of equal periodic deposits. The periodic payments can be made annually, semi-annually, quarterly, monthly,etc. The amount of the annuity is the sum of all the deposits made PLUS the interest earned on those deposits. The interest earned on those deposits is compounded interest. With annuities we are sometimes interested in the future value of the annuity or sometimes we are interested in the present value of the annuity. Section 5.3 deals with the future value of an annuity. With the future value of an annuity deposits are made and the account grows. The equation used for future value of an annuity is:

Equation #6: Future Value of an Annuity

$$A = P * \frac{(1+i)^n - 1}{i}$$

where:

A = the present value of the annuity – this is the sum of the deposits PLUS the interest earned on those deposits.
P = the amount of the deposit or payment for each payment period
i = $\frac{r}{m}$ the interest rate for the compound period is calculated by dividing the annual interest rate by the number of compound periods in a year.
n = the number of deposits made for the duration of the annuity (m * t)
m= the number of compound periods in a year
t = length of the annuity in years

Example: Ian pays $150 every month into an annuity paying 6% compounded monthly. What is the value of his annuity after 30 deposits?

Use equation #6, where we are solving for A – the amount of the annuity, P= the amount of the deposits = $150, i = .06/12, and n = 30.

$$A = P * \frac{(1+i)^n - 1}{i} = 150 * \frac{(1+(\frac{.06}{12}))^{30} - 1}{(\frac{.06}{12})} = \$4,842.00$$

The amount of interest earned on the deposits can also be calculated. If the deposits did not earn any interest the amount in the account after 30 deposits would be 30 *150 or $4,500, so the interest earned on the deposits = A-nP = 4,842.00-4,500 = $342

<u>Sinking Funds</u> – If a person knows they will owe a certain debt in the future they may decide to deposit a certain amount of money into an account (monthly, semi-annually, or annually) that is accumulating interest (compounded interest) on the payments in order to have the amount of money needed to settle their debt by a certain time period. Problems with sinking funds usually require you to find the amount of the deposit required to meet a certain debt. This is the P value in the future value of an annuity equation. You can use the future value of an annuity equation – plug-in what you know and then solve for P or you can re-arrange the equation for the future value of an annuity to solve it for P as shown in Equation #7.

Equation #7 – Future Value of an Annuity solved for P-payment (use with Sinking Fund problems)

$$P = A\left[\frac{i}{((1+i)^n - 1)}\right]$$

Example: The ABC company needs to provide for the payment of a $500,000 debt maturing in 7 years. Contributions to the fund are made quarterly and interest on the account is compounded quarterly at 10%. Find the amount necessary to deposit on a quarterly basis so the company can pay their debt.

For this problem you can use the annuity equation and solve for P – the amount of the deposit. A = $500,000 – the amount of the annuity you need in seven years, i = (.10/4) because interest is compounded monthly, and n = 4*7 = 28.

$$A = P * \frac{(1+i)^n - 1}{i} =$$

$$500000 = P * \frac{(1+(\frac{.10}{4}))^{28} - 1}{(\frac{.10}{4})} =$$

$$500000 = P * 39.86$$

$$P = \$12,543.90$$

For more examples of annuities and sinking funds look in Section 5.3 of your book – you should be able to do the homework problems on pages 288 – 290.

<u>Present Value of an Annuity – Amortization (Section 5.4)</u>

The present value of an annuity can be thought of as the amount of money needed today to invest at ***i*** percent interest such that ***n*** number of withdrawals can be made over a period of time at the end of which there will not be any money in the account

The equation for present value of an annuity is:

Equation # 8:

$$V = P * \frac{1-(1+i)^{-n}}{i}$$

where:

V= the present value of an annuity = the amount of money you need now or owe now
P = the amount of the deposit or payment for each payment period
i = $\frac{r}{m}$ the interest rate for the compound period is calculated by dividing the annual interest rate by the number of compound periods in a year.
n = the number of deposits made for the duration of the annuity (m * t)
m= the number of compound periods in a year
t = length of the annuity in years

Mr. Smith at the age of 75, can expect to live for 15 more years. If he can invest at 12% per annum compounded monthly, **how much does he need now** to guarantee himself $500 every month for the next 15 years?

For this problem you need to use the equation for present value and solve for V. P = the amount withdrawn periodically = $500, I = .12/12 (interest is compounded monthly), and n = (15*12) (monthly payments for 15 years)

$$V = P * \frac{1-(1+i)^{-n}}{i} = 500 * \frac{1-(1+(\frac{.12}{12}))^{-(15*12)}}{(\frac{.12}{12})} = \$41,660.83$$

Now try problem # 15 on page 299.

We can also have problems were we know the present value of the annuity and want to find P. These problems usually consist of a present value loan of V dollars and a periodic payment of P. After making ***n*** payments your loan is paid-off, because the loan is an annuity your payments include payments towards the original loan PLUS interest – the cost for borrowing money. This is also known as **amortization**. When you amortize a loan you make periodic payments on the loan until the balance is zero or the debt has been retired. For instance if you borrowed $20,000 to buy a car, you can amortize the loan over a period of 4 years by making monthly payments. The amount of those monthly payments is what you would solve for, to do this, take the equation for present value of an annuity and solve for P:

Equation #9 – Amortization Equation

$$P = V * \left[\frac{i}{(1-(1+i)^{-n}} \right]$$

This is known as the amortization equation and you use this equation when trying to calculate required monthly payments to pay off loans such as car loans, mortgage payments, credit cards, determining inheritance payments, and determining retirement income.

Example: Jacquie and Jim want to buy their dream house. If they make a down payment of $75,000 on a house that costs $650,000 and the mortgage rate is 7.5% with a term of 30 years, what will there monthly payment be?

Use Equation #8, the amortization equation and solve for P. V = the amount of the loan = $650,000-75,000 = $575,000 , i = .075/12, and n = 30*12 = 360.

$$P = V * \left[\frac{i}{(1-(1+i)^{-n}} \right] = 575000 * \left[\frac{\left(\frac{.075}{12}\right)}{1-\left(1+\left(\frac{.075}{12}\right)\right)^{-360}} \right] = \$4,020.48$$

What is the total amount of interest paid?
This is equal to what they pay over thirty years (n*P) minus the amount of the original loan (V):

$$nP - V = (360 * 4020.48) - 575000 = \$872,372.80$$

If Jacquie and Jim decide to pay-off their mortgage after 10 years what amount do they owe?

The balance of the unpaid loan is equal to the PRESENT VALUE of the remaining payments. Each payment equals $4,020.48 and after ten years of payments Jacquie and Jim still have to make 240 payments (20 years of payments). To find the present value we use **Equation #8**:

$$V = P\left[\frac{1-(1+i)^{-n}}{i} \right] = 4020.48 * \left[\frac{1-\left(1+\frac{.075}{12}\right)^{-240}}{\frac{.075}{12}} \right] = \$499,071$$

How much interest will Jacquie and Jim pay if they make ten years of payments and then pay the house off?

First find out how much they pay – ten years of payments plus the present value of the remaining 240 payments = 10*12*4,020.48 + 499,071 = $981,529. The amount of interest is what they paid minus the original amount of the loan or 981,529 – 575,000 = $406,529. So they save $872,372.80 – 406,529 = $465,843.80 in interest by paying the loan off in ten years instead of 30.