Math 130

Sample Test #2

Continuity

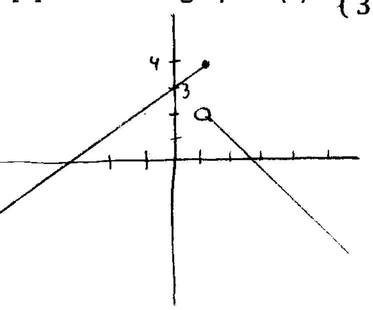
[1] Determine where the function $y = \frac{1}{x-2}$ is continuous at (a) x=0 (b) x=1 (c) x=2. If the function is discontinuous at either of the points, what type of discontinuity is it?

(a) continuous

(b) continuous

(c) discontinuous – infinite discontinuity

[2] Sketch the graph: h(x) = $\begin{cases} 3+x & if\ x \leq 1 \\ 3-x & if\ 1 < x \end{cases}$ Is h(x) continuous at 1? Why or why not?



not continuous at 1 because $\lim_{x \to 1} \nexists$ even though $f(1) = 4$

(jump discontinuity)

[3] If $f(x) = \{|x-3|\ \ if\ x \neq 3\}$ and f(x) is continuous at $x = 3$, what must f(3) equal?

$= 3-3$

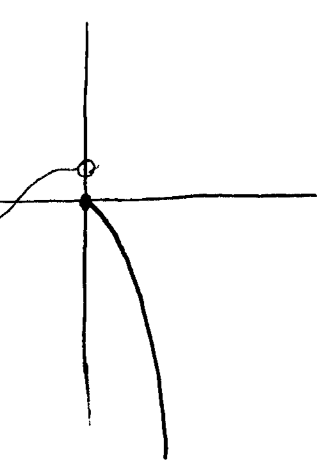
$= 0$

$f(3) = 0$

[4] If $f(x) = \begin{cases} \cos(x) & if\ x < 0 \\ 0 & if\ x = 0 \\ 1-x^2 & if\ x > 0 \end{cases}$

Where is x discontinuous and why? discontinuous at $x = 0$ (jump discontinuity) $\lim_{x \to 0} f(x) \neq f(0)$

At which points is f continuous from the right, from the left, or neither? Sketch the graph.



continuous from right : $1-x^2$

continuous from left : not continuous

neither : $x = 0$

Infinite Limits

[5] Find the limit of the following problems

(a) $\lim_{x\to\infty} \frac{2-3y^{2}}{5y^{2}+4y} = -\frac{3}{5}$

(b) $\lim_{x\to\infty} \frac{x^2+2x}{x^{3}+3x^2} = 0$

(c) $\lim_{t\to\infty} \frac{6x^4+x^{7}+3}{4x^4+3x^2+2} = \infty$

Horizontal and Vertical Asymptotes

[6] Find the horizontal and vertical asymptotes for the following equations

(a) $y = \frac{1+2x^{2}}{x^{2}-1}$ vertical : $x^2-1=0$ horizontal: 2

$(x+1)(x-1)$

$x=-1, 1$

(b) $y = \frac{-8x-4}{2x^2-7x-4}$ vertical : $2x^2-7x-4=0$ horizontal: 0

$(2x+1)(x-4)=0$

hole @ $-\frac{1}{2}$ $x=4$

(c) $y = \frac{x^2-4}{x-2} = \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$ $x=2$ (hole)

Derivatives

[7] Find the derivative of $f(x) = \frac{1}{2x-1}$ using the definition of the derivative

$$\lim_{h\to 0} \frac{\frac{1}{2(x+h)-1} - \frac{1}{2x-1}}{h} = \frac{(2x+2h-1)(2x-1)\left[\frac{1}{2(x+h)-1} - \frac{1}{2x-1}\right]}{h} = \frac{\frac{\cancel{2x}\cancel{-1}-(\cancel{2x}+2h\cancel{-1})}{(2x+2h-1)(2x-1)}}{\cancel{h}} = \frac{-2}{(2x+2h-1)(2x-1)}$$

$$\lim_{h\to 0} = \frac{-2}{(2x-1)^2}$$

[8] Using the definition of a derivative, find $f'(x)$ when $f(x) = 2x^2 + 3x$

$$\lim_{h\to 0} \frac{(2(x+h)^2+3(x+h))-(2x^2+3x)}{h} = \frac{2(x^2+2xh+h^2)+3x+3h-2x^2-3x}{h} = \frac{\cancel{2x^2}+4xh+2h^2+\cancel{3x}+3h\cancel{-2x^2}\cancel{-3x}}{h}$$

$$\lim_{h\to 0} = \frac{4xh+2h^2+3h}{h}$$

$$\lim_{h\to 0} = \frac{\cancel{h}(4x+2h+3)}{\cancel{h}}$$

$$\lim_{h\to 0} = 4x+2\cancel{h}^{0}+3$$

$$= 4x+3$$

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[9] Differentiate the following:

(a) $f(x) = 4x^7 + 16x^2 - 2\sqrt{x} + \frac{5}{x}$

$f'(x) = 28x^6 + 32x - x^{-1/2} - 5x^{-2}$

$= 28x^6 + 32x - \frac{1}{\sqrt{x}} - \frac{5}{x^2}$

(b) $y = (2x^3 - 5x^4 + 4)^5$

$y' = 5(2x^3 - 5x^4 + 4)^4(6x^2 - 20x^3)$

(c) $y = \sqrt{3x^2 + 4x + 6}$

$y' = \frac{1}{2}(3x^2 + 4x + 6)^{-\frac{1}{2}}(6x + 4)$

(d) $f(x) = (x^2 - 3x)(4x + 5)$

$f'(x) = (2x - 3)(4x + 5) + (x^2 - 3x)(4)$

$= 8x^2 + 10x - 12x - 15 + 4x^2 - 12x$

$= 12x^2 - 14x - 15$

(e) $y = (x^2 + 1)^3(x^3 - 1)^2$

$y' = (3(x^2+1)^2(2x))((x^3-1)^2) + ((x^2+1)^3)(2(x^3-1)(3x^2))$

$= (6x(x^2+1)^2)(x^3-1)^2 + (x^2+1)^3(6x^2(x^3-1))$

(f) $f(x) = \frac{x^2+1}{1-3x}$

$f'(x) = \frac{(1-3x)(2x) - (x^2+1)(-3)}{(1-3x)^2} = \frac{2x - 6x^2 + 3x^2 + 3}{9x^2 - 6x + 1} = \frac{-3x^2 + 2x + 3}{9x^2 - 6x + 1}$

(g) $f(x) = \frac{x^2-6x+9}{x-3}$

$f'(x) = \frac{(x-3)(2x-6) - (x^2-6x+9)(1)}{(x-3)^2} = \frac{(2x^2 - 6x - 6x + 18) - (x^2 - 6x + 9)}{x^2 - 6x + 9} = \frac{x^2 - 6x + 9}{x^2 - 6x + 9} = 1$

[10] Find the equation of the tangent line to the function $y = \frac{4x}{x+2}$ at the point (2, 4). Write your answer in the $y = mx + b$ form.

$y' = \frac{(x+2)(4) - (4x)(1)}{(x+2)^2} = \frac{4x + 8 - 4x}{(x+2)^2} = \frac{8}{(x+2)^2}$

$y'(2) = \frac{8}{(2+2)^2} = \frac{8}{16} = \frac{1}{2}$

$y = mx + b$

$4 = \frac{1}{2}(2) + b$

$3 = b$

$y = \frac{1}{2}x + 3$

[11] Find the points on the curve y = $3x^3 + 4.5x^2$ + 2x + 8 where the tangent is horizontal.

$y' = 9x^2 + 9x + 2$

$0 = 9x^2 + 9x + 2$

$0 = (3x+1)(3x+2)$

$3x + 1 = 0$

$3x = -1$

$x = -\frac{1}{3}$

$3x + 2 = 0$

$3x = -2$

$x = -\frac{2}{3}$

[12] Find the derivative of the following logarithmic functions:

(a) $f(x) = log_2(1-3x)$

$= \frac{\ln(1-3x)}{\ln 2} = \ln(1-3x) - \ln 2$

$= \frac{1}{1-3x}(-3) = \boxed{\frac{-3}{(1-3x)\ln 2}}$

(b) $f(x) = ln\frac{(2t+1)^3}{(3t-1)^4}$

$= \ln(2t+1)^3 - \ln(3t-1)^4$

$= 3\ln(2t+1) - 4\ln(3t-1)$

$= (3)\frac{2}{2t+1} - (4)\frac{3}{3t-1} = \frac{6}{2t+1} - \frac{12}{3t-1}$

(d) $f(x) = \frac{lnx}{x^2}$

$= \frac{(x^2)(\frac{1}{x}) - (\ln x)(2x)}{x^4} = \boxed{\frac{x - 2x\ln x}{x^4}}$

(e) $f(x) = log_5(xe^x)$

$= \frac{\ln xe^x}{\ln 5} = \frac{1}{\ln 5}(\ln xe^x) = \frac{1}{\ln 5}(\ln x + \ln e^x) = \boxed{\frac{1}{\ln 5}(\frac{1}{x} + 1)}$

(f) $f(x) = \ln(e^{-x} + xe^{-x})$

$= \ln e^{-x} + \ln x e^{-x}$

$= \ln e^{-x} + \ln x + \ln e^{-x} = -x + \frac{1}{x} + -x = \boxed{-2x + \frac{1}{x}}$

(g) $f(x) = x^2\ln(2x)$

$= 2x\ln(2x) + x^2(\frac{2}{x})$

(h) $f(x) = [\ln(1+e^x)]^2$

$= 2(\ln(1+e^x))(\frac{e^x}{1+e^x})$

$= \frac{2e^x}{1+e^x}(\ln(1+e^x))$

[13] A particle moves according to a law of motion $s = 2t^2 - 5t + 2$, $t \geq 0$, where t is measured in seconds and s in feet.

(a) Find the velocity at time t = $4t - 5$

(b) What is the velocity at time t = 1? -1

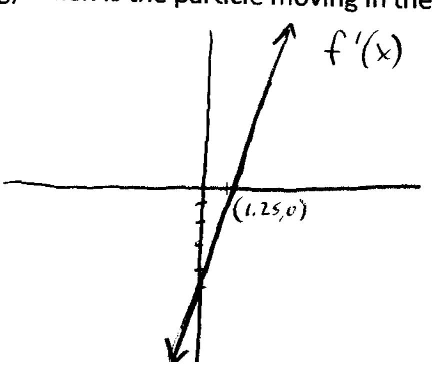
(c) What is the particle's acceleration at time t? 4

(d) What is the acceleration at 4 seconds? 4

(e) What is the speed at 1 second? 1

(f) When is the particle at rest? $4t - 5 = 0$ $t = 5/4$

(g) When is the particle moving in the positive direction? $t > 1.25$



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[14] Suppose that the cost in dollars for a company to produce x pairs of a new line of jeans is $c(x) = 1500 + 2x + .01x^2 + .0003x^3$. Find:

(a) The marginal cost of the function $c'(x) = 2 + .02x + .0009x^2$

(b) Find $c'(53)$ $= 2 + 1.06 + 2.5281$ $= 5.59$. What does it predict? \$5.59 to make the 53th pairs

(c) What will it cost to introduce the 53rd pair? \$5.53

$C(52) = 1500 + 2(52) + .01(52)^2 + .0003(52)^3$
$= 1673.2224$

$C(53) = 1500 + 2(53) + .01(53)^2 + .0003(5$
$= 1678.7531$

$c(53) - c(52) = 5.53$

[15] Find an equation of the tangent line to the graph of each equation at the specified point.

(a) $y = \sin x + \sin^2 x$ (0,0)

$y' = \cos x + 2\sin x \cos x$

$@ x = 0 \quad y'(0) = 1$

$y = mx + b$
$0 = (1)(0) + b$
$0 = b$

$y = x$

(b) $y = (1 + 2x)^{10}$ (0,1)

$y' = 10(1+2x)^9(2)$
$= 20(1+2x)^9 \quad @ x = 0 \quad y'(0) = 20$

$y = mx + b$
$1 = 20(0) + b$
$1 = b$

$y = 20x + 1$

[16] Find f', f'', f'''

(a) $f(x) = x^5 - 3x^4 + x^2 + 2$

$f'(x) = 5x^4 - 12x^3 + 2x$

$f''(x) = 20x^3 - 36x^2 + 2$

$f'''(x) = 60x^2 - 72x$

(b) $f(x) = \frac{1}{2}x^7 - \frac{4}{5}x^6 - 2x^3 + 1$

$f'(x) = \frac{7}{2}x^6 - \frac{24}{5}x^5 - 6x^2$

$f''(x) = 21x^5 - 24x^4 - 12x$

$f'''(x) = 105x^4 - 96x^3 - 12$

(c) $f(x) = \frac{3}{x^3}$ $(3x^{-3})$

$f'(x) = -9x^{-4}$

$f''(x) = 36x^{-5}$

$f'''(x) = -180x^{-6}$

(d) $f(x) = \frac{6}{\sqrt{x}}$ $(6x^{1/2})$

$f'(x) = 3x^{-1/2}$

$f''(x) = -\frac{3}{2}x^{-3/2}$

$f'''(x) = \frac{9}{4}x^{-5/2}$