

This test consists of 4 problems on 7 pages. You must show your work to receive full credit. Be sure to clearly indicate your answers. Cross out or erase any work that you do not want to be graded. You may use a scientific or graphing calculator. You are allowed one note card.

Name: Answer Key

Question	Points	Score
1	20	
2	10	
3	35	
4	35	
Total:	100	

1. The board of a major credit card company requires that the mean wait time for customers when they call customer service is at most 3.00 minutes. To make sure that the mean wait time is not exceeding the requirement, an assistant manager tracks the wait times of 45 randomly selected calls. The mean wait time was calculated to be 3.40 minutes. The population standard deviation is 1.45 minutes. The assistant manager performs a hypothesis test to test the claim that the mean wait time for customers is longer than 3.00 minutes with a significance level of $\alpha = 0.02$.

(a) [5 points] Identify the parameter of interest.

μ = mean wait time of all customers

(b) [10 points] State the null and alternative hypotheses.

$$H_0: \mu = 3.00$$

$$H_a: \mu > 3.00$$

(c) [5 points] What is the probability of a Type I Error? That is, what is the probability of rejecting the null hypothesis when it is actually true?

$$\begin{aligned} \text{Probability of Type I Error} &= \text{Significance Level} \\ &= 0.02 \end{aligned}$$

2. [10 points] The Carolina Tobacco Company advertised that its best-selling nonfiltered cigarettes contain 40 milligrams of nicotine or less. To test this claim *Consumer Advocate* magazine ran tests on 10 cigarettes from a single pack of cigarettes and calculated the mean amount of nicotine as well as the standard deviation. The amount of nicotine in cigarettes is not normally distributed. Explain why a t test **cannot** be used to test the claim that the mean amount of nicotine of the cigarettes is greater than 40 milligrams.

1. Since the cigarettes come from the same pack the sample may not be representative of the population (it is not an SRS)
2. The population is not normal and the sample size is less than 30

3. A manufacturer must test that his bolts are 2.00 cm long when they come off the assembly line. He must recalibrate his machines if the bolts are too long or too short. After sampling 100 randomly selected bolts off the assembly line, he calculates the sample mean to be 1.90 cm. He knows that the population standard deviation is 0.50 cm. Assuming a level of significance of $\alpha = 0.05$, is there sufficient evidence to show that the manufacturer needs to recalibrate the machines?

Let μ be the mean width of all of the manufacturer's bolts. The hypotheses are

$$H_0 : \mu = 2.00$$

$$H_a : \mu \neq 2.00$$

- (a) [5 points] State the question you are trying to answer.

Does the manufacturer need to recalibrate the machines?

- (b) [10 points] Determine which test you should use. Justify your answer.

Z-test:

1. SRS

2. Sample size is large ($n \geq 30$)

3. σ is known

- (c) [5 points] Calculate the test statistic.

$$\begin{aligned} Z &= \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \\ &= \frac{1.90 - 2.00}{0.50/\sqrt{100}} \\ &= -2 \end{aligned}$$

(d) [5 points] Calculate the P -value.

$H_a: \mu \neq 2.00 \leftarrow$ two-sided test

$$\begin{aligned} P &= 2 \cdot P(Z \geq 1.21) \\ &= 2 \cdot [1 - P(Z \leq 1.21)] \\ &= 2 \cdot [1 - 0.8872] \\ &= 2 \cdot 0.1128 \\ &= 0.2256 \end{aligned}$$

(e) [5 points] Should you reject H_0 or fail to reject H_0 ? Justify your answer.

$$\alpha = 0.05$$

Since $P < \alpha$, we reject H_0

(f) [5 points] Use a complete sentence to answer the original question.

There is strong evidence that the mean length of a bolt is different from 2.00 cm, so the manufacturer should recalibrate his machines.

4. When birth weights were recorded for a simple random sample of 11 male babies born to mothers in a region taking a special vitamin supplement, the sample had a mean of 3.676 kilograms and a standard deviation of 0.663 kilogram. Assume that the birth weights are coming from a normally distributed population. Use a 0.05 significance level to test the claim that the mean birth weight for all male babies of mothers given vitamins is greater than 3.38 kilograms, which is the mean for the population of all males in this particular region. Based on these results, does the vitamin supplement appear to increase the birth weight?

Let μ be the mean birth weight of all male babies born to mothers in a region taking a special vitamin supplement. the hypotheses are

$$H_0 : \mu = 3.38$$

$$H_a : \mu > 3.38$$

- (a) [5 points] State the question you are trying to answer.

Does the vitamin supplement appear to increase the birth weight?

- (b) [10 points] Determine which test you should use. Justify your answer.

t test:

1. SRS
2. Population is normal
3. σ is not known

- (c) [5 points] Calculate the test statistic.

$$\begin{aligned} t &= \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \\ &= \frac{3.676 - 3.38}{0.663/\sqrt{11}} \\ &= 1.481 \end{aligned}$$

(d) [5 points] Calculate the P -value.

$$H_a: \mu > 3.38 \quad \leftarrow \text{one-sided test}$$

$$df = n - 1 = 11 - 1 = 10$$

$$1.372 < t < 1.812 \quad \text{so} \quad 0.05 < p < 0.10$$

(e) [5 points] Should you reject H_0 or fail to reject H_0 ? Justify your answer.

$$\alpha = 0.05$$

Since $p > \alpha$ we fail to reject H_0

(f) [5 points] Use a complete sentence to answer the original question.

There is not strong evidence to support the claim that the vitamin supplement increases birth weight.