This test consists of 5 problems on 7 pages. You must show your work to receive full credit. Be sure to clearly indicate your answers. Cross out or erase any work that you do not want to be graded. You may use a scientific or graphing calculator. You are allowed one note card.

Name: Solutions

Question	Points	Score
1	10	
2	15	
3	15	
4	30	
5	30	
Total:	100	

1. The lifetimes of projector bulbs of a particular type are normally distributed with a mean of 470 hours and a standard deviation of 15 hours.

   (a) [5 points] What percentage of projector bulbs have a lifetime of between 440 and 500 hours? Justify your answer.

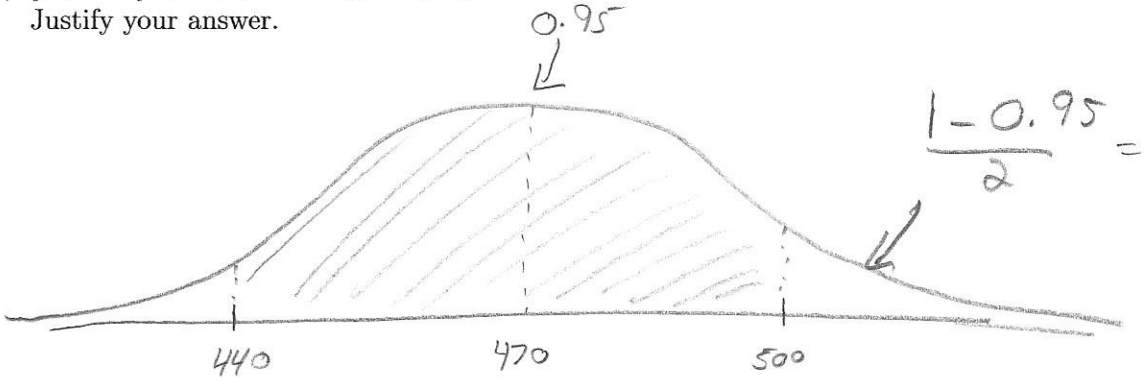
$\mu = 470$ and $\sigma = 15$.

Notice that $\mu - 2\sigma = 470 - 30 = 440$

$\mu + 2\sigma = 470 + 30 = 500$

By the 68-95-99.7 Rule, 95% of projector bulbs have a lifetime between 440 and 500 hours.

   (b) [5 points] What percentage of projector bulbs have a lifetime of more than 500 hours? Justify your answer.



2.5% of projector bulbs have a lifetime of more than 500 hours.

2. [15 points] Automated manufacturing operations are quite precise but still vary, often with distributions that are close to normal. The width in inches of slots cut by a milling machine are normally distributed with a mean of 0.8750 inches and a standard deviation of 0.0012 inches. The specifications allow slot widths between 0.8720 and 0.8780 inches. What proportion of slots meet these specifications?

$\mu = 0.8750$

$\sigma = 0.0012$

Proportion between 0.8720 and 0.8780 =

(Proportion below 0.8780) − (Proportion below 0.8720)

$$Z = \frac{x-\mu}{\sigma} = \frac{0.8780 - 0.8750}{0.0012} = 2.5$$

Proportion below 0.8780 is 0.9938

$$Z = \frac{x-\mu}{\sigma} = \frac{0.8720 - 0.8750}{0.0012} = -2.5$$

Proportion below 0.8720 is 0.0062

Proportion of slot widths between 0.8720 and 0.8780 inches is $0.9938 - 0.0062 = 0.9876$

$= 98.76\%$

3. A bag contains 5 red candies, 15 blue candies, and 10 yellow candies.

(a) [5 points] What is the probability of drawing a red candy?

$$P(\text{Red}) = \frac{5}{30} = \frac{1}{6}$$

(b) [5 points] What is the probability of drawing a red or yellow candy?

$$P(\text{Red or Yellow}) = P(\text{Red}) + P(\text{Yellow})$$

$$= \frac{5}{30} + \frac{10}{30}$$

$$= \frac{1}{6} + \frac{1}{3}$$

$$= \frac{1}{2}$$

(c) [5 points] What is the probability of drawing something besides a yellow candy?

$$P(\text{Not Yellow}) = 1 - P(\text{Yellow})$$

$$= 1 - \frac{10}{30}$$

$$= 1 - \frac{1}{3}$$

$$= \frac{2}{3}$$

4. The reading speed of second grade students is approximately normal, with a mean of 90 words per minute and a standard deviation of 10 words per minute.

   (a) [15 points] What is the probability that a student randomly selected from the population of all second grade students will read less than 85 words per minute?

Let X be the wpm of a randomly selected student.

$$P(X < 85) = P\left(Z < \frac{85-90}{10}\right)$$

$$= P(Z < -0.5)$$

$$= 0.3085$$

The probability a randomly selected student will read less than 85 wpm is 0.3085

   (b) [15 points] 24 students are selected at random from the population of all second grade students. Let $\bar{x}$ be the average words per minute of these 24 students. What is the probability that $\bar{x}$ is less than 85 words per minute?

$$P(\bar{X} < 85) = P\left(Z < \frac{85-90}{10/\sqrt{24}}\right)$$

$$= P(Z < -2.45)$$

$$= 0.0071$$

The probability the average wpm of 24 randomly selected students is less than 85 wpm is 0.0071.

5. A battery company wants to determine the mean lifetime of its AAA batteries. 25 AAA batteries are chosen at random and tested. The average lifespan of these 25 batteries is 8 hours with a standard deviation of 1 hour.

(a) [5 points] What number should be used to estimate μ, the true population mean lifespan of all AAA batteries?

$\bar{x} = 8$

(b) [5 points] What number should be used to estimate σ, the true population standard deviation lifespan of all AAA batteries?

$s = 1$

(c) [5 points] What is the standard deviation of $\bar{x}$, the mean lifespan of a random sample of 25 AAA batteries?

$\frac{s}{\sqrt{n}} = \frac{1}{\sqrt{25}} = 0.2$

(d) [5 points] What is the margin of error E of the 95% confidence interval for μ?

$E = 2 \cdot \frac{s}{\sqrt{n}} = 2 \cdot 0.2 = 0.4$

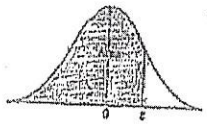
(e) [5 points] What is the 95% confidence interval for μ?

$\bar{x} \pm E$

$8 \pm 0.4 \Rightarrow 7.6$ to 8.4

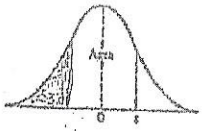
(f) [5 points] Give an interpretation for this 95% confidence interval?

We are 95% confident the true population mean μ is between 7.6 and 8.4



Area Under a Normal Curve to the Left of z, where $z = \frac{x - \mu}{\sigma}$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998



Area Under a Normal Curve to the Left of z, where $z = \frac{x - \mu}{\sigma}$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641