This test consists of 5 problems on 9 pages. You must show your work to receive full credit. Be sure to clearly indicate your answers. Cross out or erase any work that you do not want to be graded. You may use a scientific or graphing calculator. You are allowed one note card.

Name: Solutions

Question	Points	Score
1	20	
2	10	
3	20	
4	25	
5	30	
Total:	105	

1. Five students took a statistics test and the scores were as follows:

74	86	92	62	86

(a) [5 points] What is the mean of the five test scores?

$$\bar{X} = \frac{74+86+92+62+86}{5}$$

$$= \frac{400}{5}$$

$$= 80$$

(b) [5 points] What is the median of the five test scores?

62 74 86 86 92

median = 86

(c) [5 points] What is the mode of the five test scores?

mode = 86

(d) [5 points] What is the standard deviation of the five test scores?

deviations from mean	(deviations from mean)^2
$74 - 80 = -6$	36
$86 - 80 = 6$	36
$92 - 80 = 12$	144
$62 - 80 = -18$	324
$86 - 80 = 6$	$+ \quad 36$
	576

$$\text{Standard deviation} = \sqrt{\frac{\text{Sum of (deviations from mean)}^2}{\#\text{ data values} - 1}}$$

$$= \sqrt{\frac{576}{4}}$$

$$= 12$$

2. Management for a chain of restaurants recorded the number of appetizers, X, ordered by tables dining. They observed that X had the following probability distribution:

Value of X	0	1	2	3 or more
Probability	0.60	0.35	0.04	0.01

(a) [5 points] What is the probability that a randomly chosen table orders *at least* one appetizer?

$P(X \geq 1) = 0.35 + 0.04 + 0.01$

$= 0.40$

OR

$P(X \geq 1) = 1 - P(X < 1) = 1 - 0.60$

$= 0.40$

(b) [5 points] What is the probability a randomly chosen table orders less than two appetizers?

$P(X < 2) = 0.60 + 0.35$

$= 0.95$

3. The annual salaries of 121 NCAA FBS football head coaches in 2014 was compiled by USA Today. The mean salary was $\mu = \$1,824,653$ and the standard deviation was $\sigma = \$1,395,915$. Assume the salaries are normally distributed.

   (a) [10 points] If a single coach is randomly selected from the population of 121 head coaches, then what is the probabilty that coach has a salary ***greater*** than \$2,500,000?

$$Z = \frac{X - \mu}{\sigma} = \frac{2,500,000 - 1,824,653}{1,395,915}$$

$$= 0.48$$

$$P(X > 2,500,000) = P(Z > 0.48)$$

$$= 1 - P(Z < 0.48)$$

$$= 1 - 0.6844$$

$$= 0.3156$$

   (b) [10 points] If 25 coaches are randomly selected from the population of 121 head coaches, then what is the probabilty that ***mean*** salary of these 25 coaches is ***greater*** than \$2,500,000?

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{2,500,000 - 1,824,653}{1,395,915/\sqrt{25}}$$

$$= 2.42$$

$$P(\bar{X} > 2,500,000) = P(Z > 2.42)$$

$$= 1 - P(Z < 2.42)$$

$$= 1 - 0.9922$$

$$= 0.0078$$

4. The "fun size" of a Snickers bar is supposed to weigh 20 grams. Because the penalty for selling candy bars under their advertised weight is severe, the manufacturer calibrates the machine so the mean weight is 20.1 grams. The quality-control engineer is concerned about the calibration. He obtains a random sample of 11 candy bars, weighs them, and finds that the sample average is $\bar{x} = 20.301$ grams and the sample standard deviation is $s = 0.64$ grams. Let μ be the mean weight of all Snickers bars made from this machine. Does the machine need to be recalibrated? That is, is the mean weight of all Snickers bars less than 20.1 grams? Use a significance level of $\alpha = 0.05$. Assume the weights of Snickers bars are normally distributed. The hypotheses are

$$\begin{aligned} H_0 : \mu &= 20.1 \\ H_a : \mu &< 20.1 \end{aligned}$$

(a) [5 points] State the question that you are trying to answer in words.

Is the mean weight of all Snickers bars made from the machine less than 20.1 grams?

(b) [5 points] Determine which test you should use. Justify your answer.

t-test:

1. SRS
2. Weights are normally distributed
3. Population s.d. σ is not known

(c) [5 points] Calculate the test statistic.

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{20.301 - 20.1}{0.64/\sqrt{11}}$$

$$= 1.042$$

(d) [5 points] Calculate the p-value.

$df = 11 - 1 = 10$

$H_a: \mu < 20.1$ ← one-sided test

$0.879 < t < 1.093$

$\Rightarrow 0.15 < P < 0.20$

(e) [5 points] Use a complete sentence to answer your question from part (a).

Fail to reject H_0. There is not strong evidence that the mean weight of all Snickers bars is less than 20.1 grams. The machine does not need to be recalibrated.

5. In a car crash experiment, cars are classified into one of four groups: subcompact, compact, midsize, and full-size. There are five different cars in each group. Head injury data (in hic, which are standard head injury condition units) for the dummies in the driver's seat are measured. The table below is the output after an ANOVA procedure was performed. Use a 0.05 significance level. Do the sample data suggest that there is a relationship between the size of a car and how safe it is?

Anova: Single Factor

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Subcompact	5	3444	688.8	48502.7
Compact	5	2979	595.8	14142.7
Midsize	5	2434	486.8	28110.2
Full-size	5	2689	537.8	23905.2

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	112625	3	37541.67	1.30966	0.305728	3.238872
Within Groups	458643.2	16	28665.2			
Total	571268.2	19				

(a) [5 points] State the question you are trying to answer.

Is there a relationship between the size of a car and how safe it is?

(b) [5 points] Explain why an ANOVA test should be used to answer your question from part (a).

We are comparing four population means

(c) [5 points] Identify the parameters of interest and state the null and alternative hypotheses.

μ_S = mean hic score for all sub compact cars

μ_C = mean hic score for all compact cars

μ_M = mean hic score for all midsize cars

μ_F = mean hic score for all full-size cars

H_0: $\mu_S = \mu_C = \mu_M = \mu_F$

H_a: Not all of μ_S, μ_C, μ_M, and μ_F are equal

(d) [5 points] What is the test statistic?

$F = 1.30966$

(e) [5 points] What is the P-value?

$P = 0.305728$

(f) [5 points] Use a complete sentence to answer the original question.

Fail to reject H_0. There is not strong evidence that hic scores for cars depend on the size of the car