Math 114 Section A
Sample Exam II – Chapter 8

Name: Key
Date: _____

1.) How many different cars can be made if there are 6 different car models, 2 interior colors, 5 exterior colors, and 2 stereo options?

$6 \times 2 \times 5 \times 2 = 120$

2.) How many different sets of answers are possible on a test with 30 true-false questions?

$2 \times 2 \times 2 \times 2 \times \_\_\_ \ldots \quad 2^{30} = 1,073,741,824$

3.) The call letters for radio stations in the United States consist of either three or four letters beginning with either *k* or *w*. How many different call letters are possible?

$2 \times 26 \times 26 + 2 \times 26 \times 26 \times 26$

$1352 + 35,152 = 36,504$

4.) How many license plates are there consisting of two letters followed by four digits if the digits cannot be repeated but the letters can?

$26 \times 26 \times 10 \times 9 \times 8 \times 7 = 3,407,040$

5.) A computer password is composed of 4 letters. How many different passwords are possible if

a.) No repeated letters are allowed?

$26 \times 25 \times 24 \times 23 = 358,800$ or $P(26,4)$

b.)The password must start with a J, end with an L, and no repeated letters are allowed?

$\underset{J}{1} \times 24 \times 23 \times \underset{L}{1} = 552$

6.) How many "words" can be formed using all the letters in the word Football?

F - 1
O - 2
T - 1
B - 1
A - 1
L - 2

$$\frac{8!}{(1! \times 2! \times 1! \times 1! \times 1! \times 2!)} = 10,080$$

7.) In how many ways can 4 fig trees, 3 pear trees, and 6 apple trees be arranged along a fence line if one does not distinguish between trees of the same kind?

Total # of trees $4+3+6=13$

$$\frac{13!}{(4! \times 3! \times 6!)} = 60,060$$

8.) The Nagy 5K Race has 25 entries. In how many ways is it possible to award prizes for the first, second, and third place? order is important Permutation

$P(25,3) = 13,800$ or $\underset{1st}{25} \times \underset{2nd}{24} \times \underset{3rd}{23}$

9.) From a group of ten people, two are to be selected to serve on a Spring Break Party Committee. How many selections are possible? order is not important Combination

$\binom{10}{2} = 45$

10.) In how many different ways can a President, Vice President, and Secretary be selected from a class of 30 people?

$P(30,3) = 24,360$

11.) There are 5 different Math books and 4 different History books. How many ways are there to arrange them on a shelf if books of the same topic must be grouped together?

$5!$ ways to arrange the Math books
$4!$ ways to arrange the History books
$2!$ ways to arrange the subjects - Math History or History Math

$\underset{Math}{5!} \times \underset{History}{4!} \times \underset{Subject}{2!} = 5760$

12.) A deli serves sandwiches with the following options: 3 kinds of bread, 5 kinds of meat, and 2 kinds of cheese. How many different sandwiches are possible, assuming that one of each category is used?

$3 \times 5 \times 2 = 30$

13.) A bowl contains fifteen balls – 8 red, 5 blue, and 2 purple. If you pick a sample of five balls from the bowl in how many ways can you pick –

a.) Your sample of five balls?

$C(15,5)$ or $\binom{15}{5} = 3003$

b.) 5 red balls?

	Red	Others
Have	8	7
Want	5	0

$\binom{8}{5} \times \binom{7}{0} = 56 \times 1 = 56$

c.) 1 red ball, 2 blue balls, and 2 purple balls?

	Red	Blue	Purple
Have	8	5	2
Want	1	2	2

$\binom{8}{1} \times \binom{5}{2} \times \binom{2}{2} = 80$

d.) At least 4 red balls? 4 or 5 red balls at least → $\geq$

	Red	Other
Have	8	7
Want	4	1

or

	Red	Other
Have	8	7
Want	5	0

$\binom{8}{4} \times \binom{7}{1} + \binom{8}{5} \times \binom{7}{0}$

$490 + 56 = 546$

14.) Cabrini is forming an activities committee of ten people. They are selecting from 15 women and 20 men.

a.) How many different committees can be formed consisting of 5 women and 5 men?

	Women	Men
Have	15	20
Want	5	5

$\binom{15}{5} \times \binom{20}{5} = 46,558,512$

b.) How many different committees can be formed consisting of at least 8 women? (just set the problem up – don't worry about calculating out the answer) ≥ 8 8 or 9 or 10 women

	Women	Men
Have	15	20
Want	8	2
Want	9	1
Want	10	0

$\binom{15}{8} \times \binom{20}{2} + \binom{15}{9} \times \binom{20}{1} + \binom{15}{10} \times \binom{20}{0}$

$1,222,650 + 100,100 + 3003$

$= 1,325,753$

15.) From a standard deck of 52 cards, how many 6-card hands will have 2 clubs and 3 hearts?

	Clubs	Hearts	Others	Total
Have	13	13	26	52
Want	2	3	1	6

$\binom{13}{2} \times \binom{13}{3} \times \binom{26}{1}$

$78 \times 286 \times 26 =$

$580{,}008$

16.) A box contains 12 batteries, of which 4 are defective. Three (3) batteries are chosen at random, what is the probability that:

a.) Exactly two batteries are defective?

	Defective	Not Defective	Total
Have	4	8	12
Want	2	1	3

$\frac{\binom{4}{2} \times \binom{8}{1}}{\binom{12}{3}} = \frac{6 \times 8}{220} = \frac{48}{220}$

$= .2182$

b.) All three batteries are good?

	Defective	Not Defective	Total
Have	4	8	12
Want	0	3	3

$\frac{\binom{4}{0} \times \binom{8}{3}}{\binom{12}{3}} = \frac{1 \times 56}{220}$

$= .2545$

c.) At least two batteries are good? 2 or 3 are good

	Defective	Not Defective	Total
Have	4	8	12
Want	1	2	3
Want	0	3	3

$\frac{\binom{4}{1} \times \binom{8}{2} + \binom{4}{0} \times \binom{8}{3}}{\binom{12}{3}}$

$\frac{4 \times 28 + 1 \times 56}{220} = \frac{168}{220}$

binomial probability $= .7636$

17.) A dice is tossed 6 times. What is the probability of rolling

$1 - p = \frac{5}{6}$

$n = 6$

$p = \frac{1}{6}$

$x = 4$

a.) Exactly four 5's?

$\binom{6}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^2 = .0080$

binompdf$(6, \frac{1}{6}, 4)$

b.) At least four 5's? 4 or 5 or 6

$n = 6$

$p = \frac{1}{6}$

$x = 4, 5, 6$

$\binom{6}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^2 + \binom{6}{5}\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right)^1 + \binom{6}{6}\left(\frac{1}{6}\right)^6\left(\frac{5}{6}\right)^0$

$.0080 + .0006 + 0 = .0086$

$1 -$ binomcdf$(6, \frac{1}{6}, 3) = .0087$

either are correct

18.) I have 15 cds in my cd collection. My collection includes 6 Elton John cds, 4 Pearl Jam cds, 2 No Doubt cds, and 3 AC/DC cds, if I select **a sample of 3 cds** from my collection

a.) In **how many ways** can I pick a **RANDOM SAMPLE** of 3 cds?

$\binom{15}{3} = 455$

b.) In how many **ORDERS** can I play any 3 of the cds?

$P(15,3) = 2730$

c.) What is the **probability** I select 1 Pearl Jam cd and 2 AC/DC cds?

	Pearl Jam	AC/DC	Others	Total
Have	4	3	8	15
Want	1	2	0	3

$$\frac{\binom{4}{1}\times\binom{3}{2}\times\binom{8}{0}}{\binom{15}{3}} = \frac{4\times3\times1}{455} = \frac{12}{455} = \boxed{.0264}$$

d.) In **how many ways** can I pick at least 2 Elton John cds?

	Elton	Others	Total
Have	6	9	15
Want	2	1	3
Want	3	0	3

$\binom{6}{2}\times\binom{9}{1} + \binom{6}{3}\times\binom{9}{0}$

$135 + 20 = \boxed{155}$

e.) What is the **probability** I pick at least 2 Elton John cds?

$$\frac{155}{\binom{15}{3}} = \frac{155}{455} = \boxed{.3407}$$

19.) LeBron James has a 35% three-point shooting percentage (this means he makes 35% of his 3 point shots). If he shoots 4 three-point shots what is the probability he (hint use binomial probability) $n=4 \quad p=.35 \quad 1-p=.65$

a.) Makes EXACTLY 2 three-point shots?

$x=2 \quad \binom{4}{2}(.35)^2(.65)^2 = \boxed{.3105}$

b.) Makes between 1 and 3 three-point shots, inclusive? 1 or 2 or 3

$\binom{4}{1}(.35)^1(.65)^3 + \binom{4}{2}(.35)^2(.65)^2 + \binom{4}{3}(.35)^3(.65)^1$

$.3845 + .3105 + .1115 = \boxed{.8065}$

c.) Makes at most 1 three-point shot? 0 or 1

$\binom{4}{0}(.35)^0(.65)^4 + \binom{4}{1}(.35)^1(.65)^3$

$.1785 + .3845 = \boxed{.5630}$

20.) Find the expected value for the random variable, x.

x	4	6	8	10
$P(x)$	0.4	0.4	0.05	0.15

$E(x) = 4(.4) + 6(.4) + 8(.05) + 10(.15) = 5.9$

21.) Mrs. Devanney draws 3 marbles from a bag containing 3 yellow and 4 white marbles. What is the expected number of yellow marbles in the sample?

X	0	1	2	3
$P(x)$	.1143	.5143	.3429	.0286

$\frac{\binom{3}{0}\binom{4}{3}}{\binom{7}{3}} = .1143 \quad \frac{\binom{3}{1}\binom{4}{2}}{\binom{7}{3}} = .5143$

$\frac{\binom{3}{2}\binom{4}{1}}{\binom{7}{3}} = .3429 \quad \frac{\binom{3}{3}\binom{4}{0}}{\binom{7}{3}} = .0286$

$E(x) = 0(.1143) + 1(.5143) + 2(.3429) + 3(.0286)$

$= \boxed{1.2859}$

22.) Solve these problems without using the calculator

a.) $0! = 1$

b.) $P(5,2)$

$\frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5\times4\times3!}{3!}$

$= 5\times4$

$= 20$

c.) $\binom{7}{4}$

$\frac{7!}{(7-4)!\,4!} = \frac{7!}{3!\times4!} =$

$\frac{7\times6\times5\times4!}{3\times2\times1\times4!} = \frac{7\times6\times5}{6}$

$= 35$

23.) The band is selling lottery tickets. There are 3000 tickets that each cost $1. First prize is $300. There are 2 second prize tickets winning $200 each. There are 7 third prizes of $100 each. What is the expected payback if Sean buys one ticket?

x	299	199	99	-1
$p(x)$	$\frac{1}{3000}$	$\frac{2}{3000}$	$\frac{7}{3000}$	$\frac{2990}{3000}$

$$E(x) = 299\left(\frac{1}{3000}\right) + 199\left(\frac{2}{3000}\right) + 99\left(\frac{7}{3000}\right) - 1\left(\frac{2990}{3000}\right)$$

$$= \frac{299}{3000} + \frac{398}{3000} + \frac{693}{3000} - \frac{2990}{3000}$$

$$= -\frac{1600}{3000} = \$-0.53$$

Permutations:

$$P(n,r) = \frac{n!}{(n-r)!}$$

Combinations:

$$\binom{n}{r} = \frac{n!}{(n-r)!r!}$$

Binomial Probability:

$$\binom{n}{x} p^x (1-p)^{(n-x)} = binompdf(n, p, x)$$