

**Set Theory – Chapter 7 :**

- 1) Let  $U = \{12, 14, 16, 18, 20, 22, 24, 26\}$ ,  $A = \{12, 14, 16\}$ ,  $B = \{12, 20, 26\}$ , and  $C = \{26\}$ .

List the members of each of the following sets, using set braces.

$A' = \{18, 20, 22, 24, 26\}$

$A \cap B = \{12\}$

$A \cup C' = \{12, 14, 16, 18, 20, 22, 24\}$

$C' = \{12, 14, 16, 18, 20, 22, 24\}$

$(A \cap B) \cup C = \{12, 26\}$

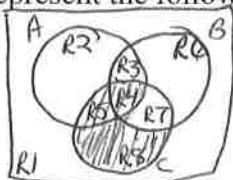
$A' \cup B' = \{14, 16, 18, 20, 22, 24, 26\}$

$B' = \{14, 16, 18, 22, 24\}$

- 2) Shade a Venn Diagram to represent the following sets:

a.  $(A \cup B') \cap C$

$\{R4, R5, R8\}$



Label your regions

$A = \{R2, R3, R4, R5\}$

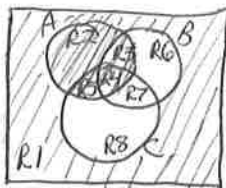
$B' = \{R1, R2, R5, R8\}$

$A \cup B' = \{R1, R2, R3, R4, R5, R8\}$

$C = \{R4, R5, R7, R8\}$

b.  $(B \cup C) \cup A$

$\{R1, R2, R3, R4, R5\}$



$B = \{R3, R4, R6, R7\}$

$C = \{R4, R5, R7, R8\}$

$B \cup C = \{R3, R4, R5, R6, R7, R8\}$

$(B \cup C)' = \{R1, R2\}$

$A = \{R2, R3, R4, R5\}$

- 3) A survey of 72 children found:

- 37 take music lessons
- 33 take dance lessons
- 29 take tennis lessons
- 12 take dance and tennis lessons
- 15 take dance and music lessons
- 13 take tennis and music lessons
- 10 take music, dance, and tennis lessons

- a. How many children take none of these types of lessons?

$72 - (19 + 5 + 10 + 3 + 16 + 2 + 14) = 3$

- b. How many children take music or dance lessons?

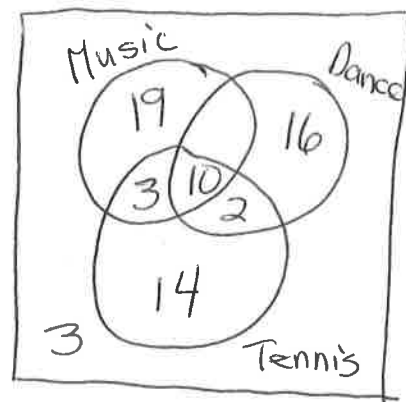
$19 + 5 + 10 + 3 + 16 + 2 = 55$

- c. How many children take music or dance lesson BUT NOT tennis lessons?

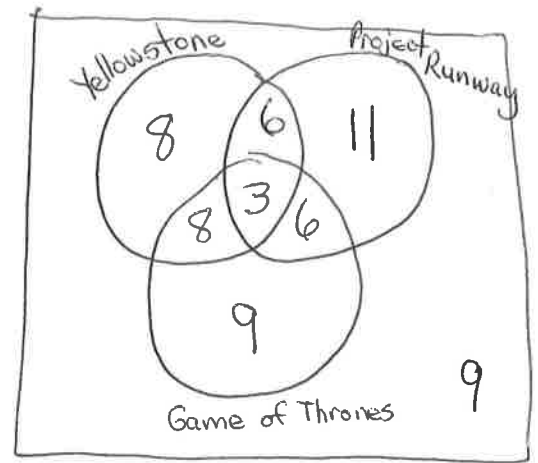
$19 + 5 + 16 = 40$

- d. How many children take only tennis lessons?

14



- 4) In a survey of 60 people, it was found that :
- 25 watch Yellowstone
  - 26 watch Project Runway
  - 26 watch Game of Thrones
  - 9 watch both Yellowstone and Project Runway
  - 11 watch both Yellowstone and Game of Thrones
  - 9 watch Game of Thrones and Project Runway
  - 3 watched all 3 shows



- a. How many people watch exactly two of the three shows?  
 $8 + 6 + 6 = 20$
- b. How many people watch exactly one of the shows?  
 $8 + 11 + 9 = 28$
- c. How many people do not watch any of these shows?  
 $60 - (8 + 6 + 3 + 8 + 11 + 6 + 9) = 9$
- d. How many people watch Project Runway or Game of Thrones BUT NOT Yellowstone  
 $11 + 6 + 9 = 26$

**Multiplication Principle, Combinations, and Permutations – Chapter 8:**

- 5) Evaluate the following expressions:

$$\begin{aligned}
 \text{a. } \frac{10!}{4!(10-4)!} &= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \times 6 \times 5} = 10 \times 3 \times 7 = 210 \\
 \text{b. } {}_8P_3 &= \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} = 8 \times 7 \times 6 = 336 \\
 \text{c. } {}_7C_4 &= \frac{7!}{(7-4)!4!} = \frac{7!}{3!4!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1} = 35
 \end{aligned}$$

- 6) How many different sets of answers are possible on a test with 8 true-false questions?  $2^8 = 256$

$$\begin{array}{cccccccc}
 \underline{2} & \underline{2} & \underline{2} & \underline{2} & \underline{2} & \underline{2} & \underline{2} & \underline{2} \\
 \text{T or F} & \text{T or F} & \text{T or F} & \text{T or F} & \text{T or F} & \text{T or F} & \text{T or F} & \text{T or F}
 \end{array}$$

- 7) A license plate is made with 4 characters. The first 2 are letters that can be repeated, the next 2 are digits that cannot be repeated. How many different license plates are possible? \_\_\_\_\_

$$\begin{array}{cccc}
 \underline{26} & \times & \underline{26} & \times & \underline{10} & \times & \underline{9} \\
 \text{L} & & \text{L} & & \text{D} & & \text{D}
 \end{array} = 60,840$$

- 8) A coffee shop offers 5 types of coffee, 3 different creamers, and 4 pastries. In how many ways can one of each be selected? 60

$$\begin{array}{ccc}
 \underline{5} & \times & \underline{3} & \times & \underline{4} \\
 \text{Coffee} & & \text{Creamer} & & \text{Pastry}
 \end{array}$$

9) How many ways can a president and vice-president be selected in a class of twenty-five students? 25 P<sub>2</sub>

$$\frac{25!}{(25-2)!} = 600$$

Permutation - order matters

10) How many ways can a three person committee be chosen from a group of six people? 6 C<sub>3</sub>

$$\frac{6!}{(6-3)!3!} = 20$$

Combination  
Order is not important

**Probability- Chapters 7 & 8:**

11) A box contains 6 white marbles, 3 red marbles, 2 black marbles, and 4 orange marbles. 15 total marbles

a. What is the probability that you pick a black marble?  $\frac{2}{15}$

b. What is the probability that you pick a white OR red marble?

$$\frac{3}{5} \qquad \frac{6+3}{15} = \frac{9}{15} = \frac{3}{5}$$

c. What is the probability you pick a marble that is not orange?

$$\frac{11}{15} \qquad \frac{6+3+2}{15} = \frac{11}{15}$$

12) A pair of dice is rolled once. Find the following probabilities for the sum of the dice:

a. P(7)  $\frac{6}{36} = \frac{1}{6}$

b. P(4)  $\frac{3}{36} = \frac{1}{12}$

c. P(7 OR 4)  $\frac{6+3}{36} = \frac{9}{36} = \frac{1}{4}$

Are these mutually exclusive events? yes

d. P(sum is even OR greater than 7)  $\frac{2}{3}$

Are these mutually exclusive events? no  $P(\text{even and } >7) = \frac{1}{4}$ , not 0

mutually exclusive  
if  $E \cap F = \emptyset$   
or  $P(E \cap F) = 0$

2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1
	36

Union Rule

$$P(\text{sum is even}) + P(\text{sum} > 7) - P(\text{sum is even and } > 7)$$

$$\frac{18}{36} + \frac{15}{36} - \frac{9}{36} = \frac{24}{36} = \frac{2}{3}$$

13) Determine if the following probabilities are valid for an event. If NOT VALID, TELL ME WHY!

a.  $P(1) = .3$   $P(2) = .7$   $P(3) = .2$   $.3 + .7 + .2 = 1.2$  (No)  $> 1$

b.  $P(1) = .75$   $P(2) = -.15$   $P(3) = .35$  (No)  $P(2) = -.15$  can't be negative

c.  $P(1) = .25$ ,  $P(2) = .35$ ,  $P(3) = .40$   $.25 + .35 + .40 = 1.0$  (yes)

14) If a single card is drawn randomly from a standard 52-card deck, find the probability that it will be a black card or a 9.

$$P(\text{black}) + P(9) - P(\text{black and } 9) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}$$

15) Given  $P(E) = .55$ ,  $P(F) = .80$ , and  $P(E \cup F) = .95$  find:

a.  $P(E \cap F) = .40$

b.  $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{.40}{.80} = .5$

c.  $P(E'|F) = \frac{P(E' \cap F)}{P(F)} = \frac{.40}{.80} = .5$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$.95 = .55 + .80 - P(E \cap F)$$

$$.95 = 1.35 - P(E \cap F)$$

$$P(E \cap F) = .40$$

	E	E'	Total
F	.40	.40	.80
F'	.15	.05	.20
Total	.55	.45	1.00

Independent  
 $P(E|F) = P(E)$   
 or

$$P(E \cap F) = P(E) \times P(F)$$

d. Are events E and F independent? No

$$P(E|F) \stackrel{?}{=} P(E) \quad P(E \cap F) \stackrel{?}{=} P(E) \times P(F)$$

$$.5 \neq .55 \quad .4 \neq .55 \times .80$$

16) 90 students are enrolled in school. 50 students get an A in MAT114. 25 students get an A in BIO101. 60 students get an A in MAT114 or BIO101.

a. What is the probability that Tyrell gets an A in both classes?  $.1667$

$$60 = 50 + 25 - n(M \cap B)$$

$$60 = 75 - n(M \cap B)$$

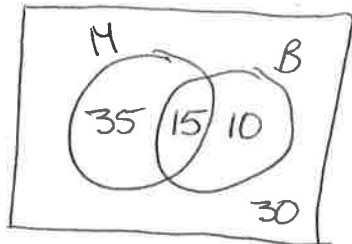
$$15 = n(M \cap B)$$

$$\frac{15}{90} = \frac{1}{6}$$

b. What is the probability that poor Tyrell doesn't get an A in either MAT114 or BIO101?  $.3333$

$$90 - 60 = 30 \quad \frac{30}{90} = \frac{1}{3}$$

c. Are these two events mutually exclusive?  
 no, they have an intersection



17) I am going on a road trip and I want to take 5 cds with me to play in my car. If my cd collection includes: 4 Elton John cds, 5 Adele cds, and 7 Beyoncé cds find:

16 total cds

a. The number of ways I can randomly select 5 cds from my collection.

$$16C_5 = 4368$$

b. What is the probability that I pick 2 Elton John cds, 1 Adele cd, and 2

	Beyoncé cds?			
Have	EJ	A	B	Total
Want	4	5	7	16
	2	1	2	5

$$\frac{\binom{4}{2} \times \binom{5}{1} \times \binom{7}{2}}{\binom{16}{5}} = \frac{6 \times 5 \times 21}{4368} = \frac{630}{4368} = .144$$

c. What is the probability I pick exactly 3 Elton John cds?

	EJ	Others	Total	
Have	4	12	16	
Want	3	2	5	

$$\frac{4C_3 \times 12C_2}{16C_5} = \frac{4 \times 66}{4368} = \frac{264}{4368} = .0604$$

d. How many orders could I play any 5 cds?

$$P(16, 5) = 16P_5 = 524,160$$

e. What is the probability I pick at least 4 Adele cds? 4 or 5

	Adele	Others	Total	
Have	5	11	16	
Want	4	1	5	
Want	5	0	5	

$$\frac{\binom{5}{4} \times \binom{11}{1} + \binom{5}{5} \times \binom{11}{0}}{\binom{16}{5}} = \frac{5 \times 11 + 1 \times 1}{4368} = \frac{56}{4368} = .0128$$

18) I flip a coin four times, find the probability that I get

success = getting a head

① Repeated fixed # of times

② only 2 outcomes - success or failure

③ Independent

so

Binomial Probability

a. All heads (4 heads)  $\binom{4}{4} (.5)^4 (1-.5)^{4-0}$

$$= .0625$$

$$n=4$$

$$p=.5$$

$$x=4$$

b. Two heads and two tails  $.375$

$$\binom{4}{2} (.5)^2 (.5)^2$$

$$n=4$$

$$p=.5$$

$$x=2$$

19) Joel Embiid has a 75% chance of making a foul shot. If he shoots 10 foul shots what is the probability that he

$$n=10$$

$$p=.75$$

$$x=6$$

a. Makes exactly 6 shots?  $\binom{10}{6} (.75)^6 (.25)^4 = .1460$

b. Makes at least 4 shots?  $.9965$

0 1 2 3 4 5 6 7 8 9 10 use complement rule

$$1 - \left[ \binom{10}{0} (.75)^0 (.25)^{10} + \binom{10}{1} (.75)^1 (.25)^9 + \binom{10}{2} (.75)^2 (.25)^8 + \binom{10}{3} (.75)^3 (.25)^7 \right]$$

$$1 - .0035 = \boxed{.9965}$$

$$n=10$$

$$p=.75$$

$$x=0, 1, 2, 3$$

20) In a class of twelve students there are 8 students wearing jeans, the rest of the student are not wearing jeans. If I decide to take three random students to the movies find the probability that

a. I take exactly 2 students wearing jeans.  $\frac{\binom{8}{2}\binom{4}{1}}{\binom{12}{3}} = \frac{28 \times 4}{220} = \frac{112}{220} = 0.5091$

b. I take at least 2 students wearing jeans.  
2 or 3  $\frac{\binom{8}{2}\binom{4}{1} + \binom{8}{3}\binom{4}{0}}{\binom{12}{3}} = \frac{112 + 56}{220} = \frac{168}{220} = 0.7636$

c. I take 0 students wearing jeans  $\frac{\binom{8}{0}\binom{4}{3}}{\binom{12}{3}} = \frac{4}{220} = 0.0182$

Have	Jeans	Jeans	Total
Want	8	4	12
	0	3	3

Jeans	No Jeans	Total
8	4	12
2	1	3

21) A university cafeteria surveyed the students who ate breakfast there for their coffee preferences. The findings are as follows:

	Do not drink coffee	Prefer regular coffee	Prefer decaf-coffee	Total
Female	60	125	20	205
Male	25	140	15	180
Total	85	265	35	385

a. Find the probability that a random person selected – does not drink coffee.

$$\frac{85}{385} = 0.2208$$

b. Find the probability that a random person selected – is male.

$$\frac{180}{385} = 0.4675$$

c. Find the probability that a random person selected – is a female who prefers regular coffee.

$$\frac{125}{385} = 0.3247$$

d. Find the probability that a random person selected – is a male, given that the student prefers decaf.

$$\frac{15}{35} = 0.4286$$

e. Find the probability that a random person selected – is female, given that the student prefers regular coffee OR does not drink coffee.

$$\frac{125 + 60}{265 + 85} = \frac{185}{350} = 0.5286$$

22) Find the expected value for the random variable:

x	0	1	2	3
P(x)	0.1	0.4	0.3	0.2

$$E(x) = 0(0.1) + 1(0.4) + 2(0.3) + 3(0.2) = 1.6$$

Statistics – Chapter 9:

23) Using the following frequency chart for ages of senators in the 103rd Congress calculate the mean, mode, and median. Hint: you will need to calculate the midpoint for each class. Midpoint  $\frac{30+39}{2} = 34.5$

Age (yrs)	Frequency	Midpoint	Product
30-39	8	34.5	276
40-49	22	44.5	979
50-59	33	54.5	1798.5
60-69	25	64.5	1612.5
70-79	9	74.5	670.5
80-89	3	84.5	253.5
Totals:	100		5590

$$\bar{x} = \frac{\sum \text{Product}}{\sum \text{Frequency}} = \frac{5590}{100} = 55.9$$

median  $\frac{100+1}{2} = 50.5$  Look at the 50<sup>th</sup> and 51<sup>st</sup> numbers. They are in the interval 50-59 so the median is its midpoint 54.5

mode 54.5 The midpoint of the interval with the highest frequency (50-59)

24) A student's quiz scores over a semester are:

8, 9, 8, 6, 8, 7, 10, 9, 11, 8, 10, 9, 5, 12

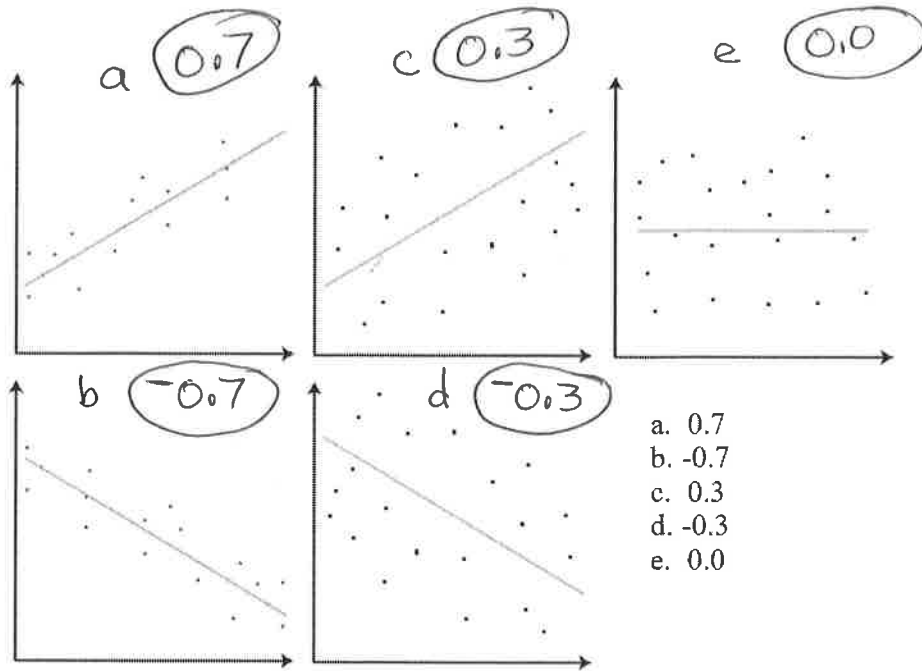
Find the Mean  $\bar{x} = \frac{120}{14} = 8.5714$

Median  $\frac{8+9}{2} = 8.5$  5, 6, 7, 8, 8, 8, 8, 9, 9, 9, 10, 10, 11, 12

Mode 8

Range  $12 - 5 = 7$

25) Select the correct correlation coefficient for each graph



26) The following data is given:

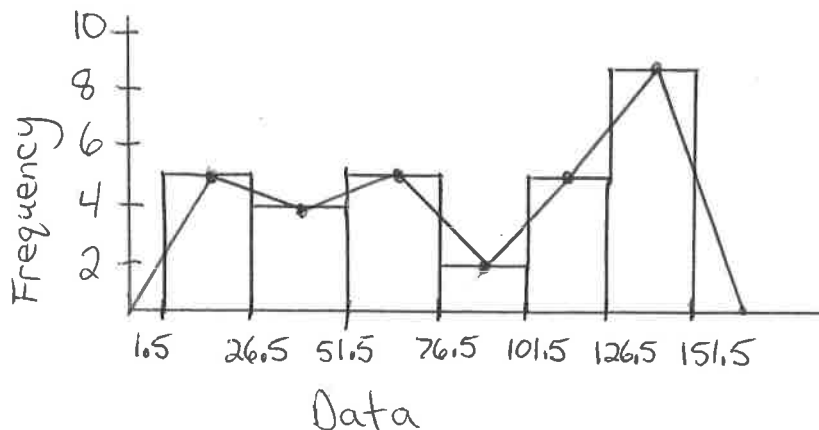
72, 135, 2, 130, 16, 34, 108, 22, 140, 117, 140, 119, 146, 135, 60, 145, 135, 71, 48, 44, 100, 51, 60, 109, 7, 137, 14, 86, 120, 63

a. Create a frequency chart with 6 classes starting with the interval 2-26

Interval	Tally	Frequency
2-26		5
27-51		4
52-76		5
77-101		2
102-126		5
127-151		9

$26-2=24$   
 $27+24=51$   
 $52+24=76$

b. Construct a histogram and frequency polygon





27) The following test scores are a random sample from all students at Cabrini taking the Math 114 final. Calculate the Mean, Range, Variance, and Standard Deviation. (This table may not be given to you on the final.)

Test Score	$\bar{x}$	$(x-\bar{x})$	$(x-\bar{x})^2$
99	82	17	289
62	82	-20	400
53	82	-29	841
85	82	3	9
89	82	7	49
95	82	13	169
82	82	0	0
80	82	-2	4
100	82	18	324
75	82	-7	49

Total: 820

⊕

2134

$$\text{Sample mean } \bar{x} = \frac{820}{10} = 82$$

$$\text{Range } 100 - 53 = 47$$

$$\text{Sample Variance } s^2 = \frac{2134}{10-1} = \frac{2134}{9} = 237.1111$$

$$\text{Sample Standard Deviation } s = \sqrt{237.1111} = 15.3984$$

What would you do differently if the data above was data from the entire population?

Population Mean

Population Variance

Population Standard Deviation

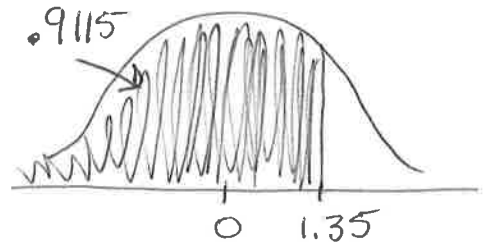
$$\mu = \frac{820}{10} = 82$$

$$\sigma^2 = \frac{2134}{10} = 213.4$$

$$\sigma = \sqrt{213.4} = 14.6082$$

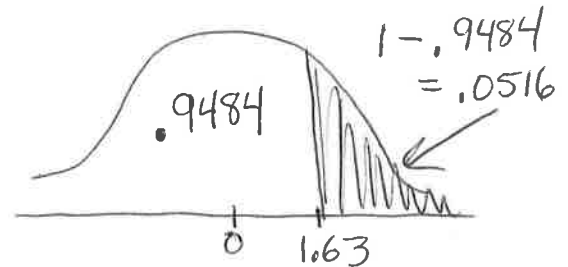
28) Using the z-score table in the back of your textbook what is the area under the normal distribution curve

a. to the left of  $z = 1.35$  .9115



b. to the right of  $z = 1.63$  .0516

$$1 - .9484 = .0516$$

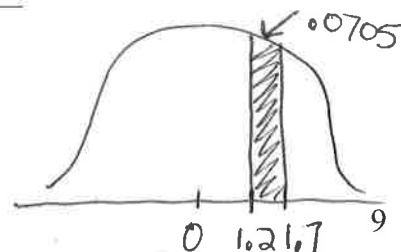


c. in between  $z = 1.20$  and  $z = 1.70$  .0705

$$z = 1.70 \quad \text{area} = .9554$$

$$z = 1.20 \quad \text{area} = .8849$$

$$.9554 - .8849 = .0705$$



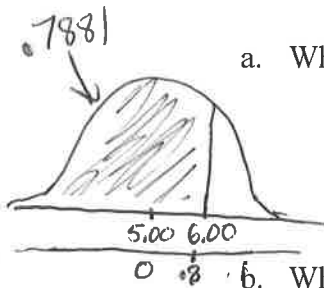
29) Given the values for population mean ( $\mu$ ) and population standard deviation ( $\sigma$ ) calculate the z values for the specified x value.

a.)  $\mu = 65, \sigma = 10, x = 72$  
$$z = \frac{72 - 65}{10} = .7$$

b.)  $\mu = 310, \sigma = 70, x = 220$  
$$z = \frac{220 - 310}{70} = -1.29$$

c.)  $\mu = 1000, \sigma = 400, x = 560$  
$$z = \frac{560 - 1000}{400} = -1.1$$

30) Customers at Starbucks spend an average of \$5.00 with a standard deviation of \$1.25. Assume this is a normal distribution, find the following:

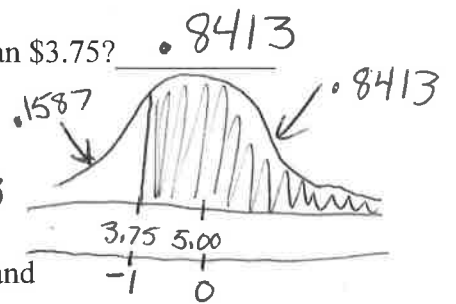


a. What percentage of customers spend less than \$6.00? 78.81%  

$$z = \frac{x - \mu}{\sigma} = \frac{6 - 5}{1.25} = .8$$
 look up .8 in the table  
 area = .7881

b. What is the probability the customer spends more than \$3.75? .8413  

$$z = \frac{3.75 - 5.00}{1.25} = -1$$
  
 area is .1587  $1 - .1587 = .8413$

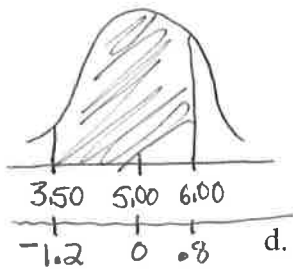


c. What percentage of customers spent between \$3.50 and \$6.00? 67.3%  

$$z = \frac{6 - 5}{1.25} = .8 \rightarrow \text{area } .7881$$
  

$$z = \frac{3.5 - 5}{1.25} = -1.2 \rightarrow \text{area } .1151$$
  

$$.7881 - .1151 = .673$$

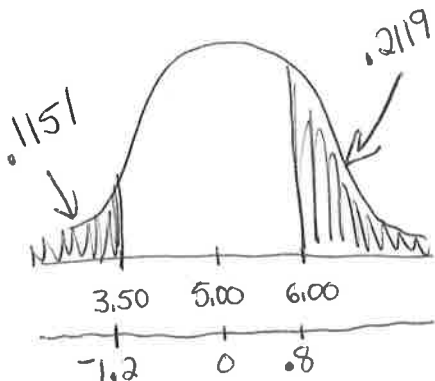


d. What percentage of students spent less than \$3.50 or more than \$6.00? 32.7%  

$$z = \frac{6 - 5}{1.25} = .8$$
  
 area to the left is .7881  

$$1 - .7881 = .2119$$
  

$$z = \frac{3.5 - 5}{1.25} = -1.2$$
  
 area to the left is .1151



Add the two areas together  

$$.2119 + .1151 = .327$$