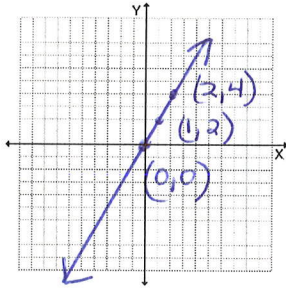


# Math 113 Practice Problems for Final Exam

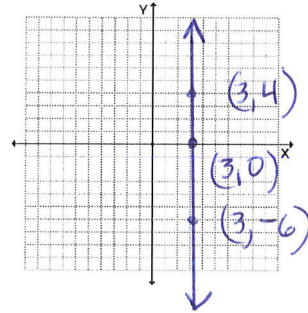
This Practice final has questions from Chapter 1, Chapter 5, Chapter 2 and then Geometry. The final may not be presented in this same order.

1. Sketch a graph of each line and identify the points you used to draw the line.

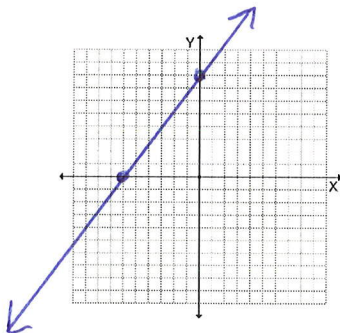
a.  $y=2x$



b.  $x=3$



c.  $-4x + 3y = 24$



x	y
0	8
	0

$$-4(0) + 3y = 24$$

$$\frac{3y}{3} = \frac{24}{3}$$

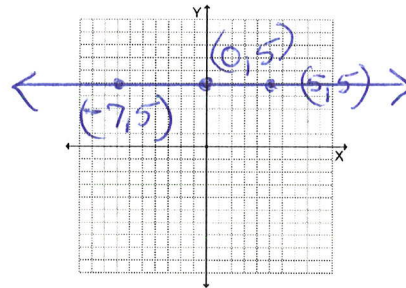
$$y = 8$$

$$-4x + 3(0) = 24$$

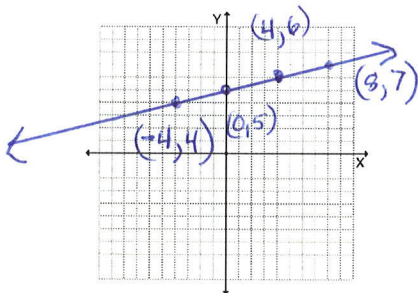
$$\frac{-4x}{-4} = \frac{24}{-4}$$

$$x = -6$$

d.  $y=5$



e.  $\frac{1}{4}x + 5 = y$



$$y = \frac{1}{4}x + 5$$

$$y = mx + b$$

2. Write an equation in slope-intercept form, if possible, for a line that:

a. Passes through the points (4,3) and (4,-5)

$$m = \frac{-5-3}{4-4} = \frac{-8}{0} \quad \text{so slope is undefined and this is a vertical line}$$

$$x = 4$$

b. Passes through the points (5,2) and (8,-3)

$$m = \frac{-3-2}{8-5} = \frac{-5}{3}$$

$$2 = \frac{-5}{3}(5) + b$$

$$2 = \frac{-25}{3} + b$$

$$\frac{6}{3} = \frac{-25}{3} + b$$

$$\frac{31}{3} = b$$

$$y = \frac{-5}{3}x + \frac{31}{3}$$

c. Is perpendicular to the line  $4x + 3y = 12$  and passes through the point (1,3)

$$4x + 3y = 12$$

$$3y = -4x + 12$$

$$y = \frac{-4}{3}x + 4$$

so, the new slope will be  $\frac{3}{4}$

$$3 = \frac{3}{4}(1) + b$$

$$3 = \frac{3}{4} + b$$

$$\frac{12}{4} = \frac{3}{4} + b$$

$$\frac{9}{4} = b$$

$$y = \frac{3}{4}x + \frac{9}{4}$$

d. Is parallel to the line  $-2x + y = -3$  and passes through the point (1,5)

$$-2x + y = -3$$

$$y = 2x - 3$$

so, the new slope will be 2

$$5 = 2(1) + b$$

$$5 = 2 + b$$

$$3 = b$$

$$y = 2x + 3$$

3. The sales of a small company were \$4,000 in 2002 and \$18,000 in 2010. Let  $y$  represent the sales in the  $x$ th year after 2002, where  $x = 0$  is the year 2002.

a. Determine a linear model that will provide the sales figures that occur in some particular year,  $x$ .

$$y = mx + b$$

$$m = \frac{18000 - 4000}{8 - 0} = \frac{14000}{8} = 1750$$

$(0, 4000)$   
 $(8, 18000)$

Now, solve for  $b$ .

$$4000 = 1750(0) + b$$

$$4000 = 0 + b$$

$$4000 = b$$

$$y = \underline{1750x + 4000}$$

b. Use your answer from part (a) to predict the amount of sales in the year 2018.

$$\begin{array}{r} 2018 \\ - 2002 \\ \hline 16 \end{array}$$

$$y = 1750(16) + 4000$$

$$y = 28000 + 4000$$

$$y = 32,000$$

$$\text{Sales in 2018} = \underline{\$32,000}$$

4. The daily fixed cost to produce widgets is \$5,750 and the cost to build a widget is \$65. Each widget sells for \$90.

- a. Find the cost equation for widgets.

$$C(x) = 65x + 5750$$

- b. Find the revenue equation for widgets

$$R(x) = 90x$$

- c. Find the profit equation for widgets

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 90x - [65x + 5750] \\ &= 90x - 65x - 5750 \\ P(x) &= 25x - 5750 \end{aligned}$$

- d. What is the number of widgets you need to sell to break even?

$$\begin{aligned} C(x) &= R(x) \\ 65x + 5750 &= 90x \\ -65x \quad & \quad -65x \\ \hline 5750 &= 25x \\ \frac{25}{25} & \quad \frac{25}{25} \\ 230 &= x \end{aligned} \quad 230 \text{ widgets}$$

- e. What is the profit if the company sells 500 widgets?

$$\begin{aligned} P(x) &= 25x - 5750 \\ &= 25(500) - 5750 \\ &= 12500 - 5750 \\ &= 6750 \end{aligned}$$

The profit is \$6,750

5. It costs a company \$760 to make 22 cell phones. It also costs the company \$220 to make 4 cell phones. The company sells the cell phones for \$55 apiece.

a. What is the cost function for the product?

$$m = \frac{220 - 760}{4 - 22} = \frac{-540}{-18} = 30$$

$$c(x) = 30x + 100$$

Solve for b

$$760 = 30(22) + b$$

$$760 = 660 + b$$

$$100 = b$$

b. What is the revenue function?

$$R(x) = 55x$$

c. What is the profit function?

$$\begin{aligned} P(x) &= R(x) - c(x) \\ &= 55x - (30x + 100) \\ &= 55x - 30x - 100 \end{aligned}$$

$$P(x) = 25x - 100$$

d. Find the break-even point.

$$R(x) = c(x)$$

$$55x = 30x + 100$$

$$25x = 100$$

$$x = 4$$

4 cell phones

e. What will the company's profit be if it makes and sells 75 cell phones?

$$\begin{aligned} P(x) &= 25x - 100 \\ &= 25(75) - 100 \\ &= 1875 - 100 \\ &= 1775 \end{aligned}$$

\$1,775

f. How many cell phones must be sold to make a profit of \$30,000?

$$\begin{aligned} P(x) &= 25x - 100 \\ 30,000 &= 25x - 100 \\ &+ 100 \qquad + 100 \end{aligned}$$

$$\frac{30,100}{25} = \frac{25x}{25}$$

$$1204 = x$$

1,204 cell phones

6. Farmer Brown is growing raspberries and is getting ready to sell them at the local farmer's market. The supply and demand functions for raspberries are

$$p = S(q) = 0.45q$$

$$p = D(q) = 4 - 0.35q$$

where  $p$  is the price per pint and  $q$  is the quantity in hundreds of pints.

- a. Find the price if the demand is 200 pints so  $q = 2$

$$p = D(q) = 4 - 0.35q$$

$$= 4 - 0.35(2)$$

$$= 4 - 0.7$$

$$= 3.30$$

$p = \$3.30$

- b. Find the demand if the price is \$1.20 per pint

$$1.20 = 4 - 0.35q$$

$$\begin{array}{r} -4 \\ -4 \end{array}$$

$$\frac{-2.8}{-0.35} = \frac{-0.35q}{-0.35}$$

$$8 = q \quad \text{so } 800 \text{ pints}$$

- c. Find the supply if the price is \$1.62 per pint

$$p = S(q) = 0.45q$$

$$\frac{1.62}{.45} = \frac{0.45q}{.45}$$

$$3.6 = q \quad \text{so } 360 \text{ pints}$$

- d. Find the equilibrium quantity and the equilibrium price

$$S(q) = D(q)$$

$$.45q = 4 - 0.35q$$

$$\begin{array}{r} + .35q \\ + .35q \end{array}$$

$$\frac{.8q}{.8} = \frac{4}{.8}$$

equilibrium  
quantity  $q = 5$   
500 pints

The price for that quantity is

$$p = S(q) = 0.45q \quad \text{or} \quad p = D(q) = 4 - 0.35q$$

$$p = 0.45(5) \quad \text{or} \quad p = 4 - 0.35(5)$$

$$p = 2.25 \quad \text{or} \quad p = 4 - 1.75$$

$$p = 2.25$$

equilibrium  
price \$2.25

7. Jane borrows \$25,000 at an interest rate of 9.25% compounded monthly to buy a new car. The terms of the loan require her to make monthly payments for the next 5 years. What will her monthly payment be?

equal payments  
amortization

R = regular payment  
P = principal  
P = 25,000  
r = .0925  
m = 12  
t = 5

$$P = R \left[ \frac{1 - \left(1 + \frac{r}{m}\right)^{-mt}}{\frac{r}{m}} \right]$$

$$25,000 = R \left[ \frac{1 - \left(1 + \frac{.0925}{12}\right)^{-(12 \times 5)}}{\frac{.0925}{12}} \right]$$

$$25,000 = R(47.89295359)$$

$$R = 521.997 = \$522.00$$

8. Susan makes yearly semi-annual deposits of \$2,500 into a savings account that earns 8.5% interest compounded semi-annually -

Equal payments  
looking for future amount  
Future Value of Annuity

R = 2500  
r = .085  
m = 2  
t = 25

a. How much money will be in the account 25 years from now?

$$S = R \left[ \frac{\left(1 + \frac{r}{m}\right)^{mt} - 1}{\frac{r}{m}} \right]$$

$$S = 2500 \left[ \frac{\left(1 + \frac{.085}{2}\right)^{2 \times 25} - 1}{\frac{.085}{2}} \right] = \$412,538.14$$

- b. How much interest will Susan earn over 25 years?

Susan deposited  $2500 \times 2 \times 25 = 125,000$   
 Future Value - Deposits = Interest Earned  
 $412,538.14 - 125,000 = \$287,538.14$

9. Marianne deposits \$5000 into an account that earns 8.75% interest. How much will be in her account in ten years if the interest is -

P = 5000  
r = .0875  
t = 10

a. Simple

$$A = P(1 + rt)$$

$$= 5000(1 + (.0875 \times 10))$$

$$= \$9,375.00$$

P = 5000  
r = .0875  
m = 4  
t = 10

b. Compounded quarterly

$$A = P \left(1 + \frac{r}{m}\right)^{mt}$$

$$= 5000 \left(1 + \frac{.0875}{4}\right)^{4 \times 10}$$

$$= \$11,881.77$$

P = 5000  
r = .0875  
m = 365  
t = 10

c. Compounded daily

$$A = P \left(1 + \frac{r}{m}\right)^{mt}$$

$$= 5000 \left(1 + \frac{.0875}{365}\right)^{365 \times 10}$$

$$= \$11,993.12$$

10. Anthony and Caitlyn are both graduates of Cabrini University. They both agree to contribute to Cabrini's Endowment Fund. Anthony says he will give \$75 at the end of each month for 10 years. Caitlyn prefers to give a lump sum today. What lump sum should she give that will equal the present value of Anthony's monthly gifts if the Endowment Fund pays 4.8 interest compounded monthly?

looking for a present value

$R = 75$   
 $r = .048$   
 $m = 12$   
 $t = 10$

$$P = R \left[ \frac{1 - \left(1 + \frac{r}{m}\right)^{-mt}}{\frac{r}{m}} \right]$$

$$P = 75 \left[ \frac{1 - \left(1 + \frac{.048}{12}\right)^{-(12 \times 10)}}{\frac{.048}{12}} \right] = \$7136.70$$

11. Brittany and Kevin estimate that they can afford to pay \$2500 for a monthly mortgage payment. If their mortgage is a 30 year mortgage at a rate of 6.4% compounded monthly:

equal payments amortization

$R = 2500$   
 $r = .064$   
 $m = 12$   
 $t = 30$

- a. How much can they afford to borrow?

$$P = R \left[ \frac{1 - \left(1 + \frac{r}{m}\right)^{-mt}}{\frac{r}{m}} \right]$$

$$P = 2500 \left[ \frac{1 - \left(1 + \frac{.064}{12}\right)^{-12 \times 30}}{\frac{.064}{12}} \right]$$

$$= \$399,676.48$$

- b. How much interest will they end up paying at the end of the 30 year mortgage?

Total paid =  $2500 \times 12 \times 30 = \$900,000$

Paid - Principal = Interest

$900,000 - 399,676.48 = \$500,323.52$  interest paid

12. Mr. and Mrs. Smith are buying their first house which has a price of \$318,000. They have a down payment of \$31,800 and will take out a 15-year mortgage for the remainder of the cost of the house. The mortgage has an annual interest rate of 3.1%.

- a. Find the amount of the monthly payment

$318,000 - 31,800 = \$286,200$  amount of loan needed (present value)

$P = 286,200$   
 $r = .031$   
 $m = 12$   
 $t = 15$

$$P = R \left[ \frac{1 - \left(1 + \frac{r}{m}\right)^{-mt}}{\frac{r}{m}} \right]$$

$$286,200 = R \left[ \frac{1 - \left(1 + \frac{.031}{12}\right)^{-12 \times 15}}{\frac{.031}{12}} \right]$$

$286,200 = R(143,801.8671)$   
 $R = \$1990.24$  monthly payment

- b. Find the total amount of interest paid

$1990.24 \times 12 \times 15 = 358,243.20$  Total paid

Total Paid - Amt Borrowed = Interest

$358,243.20 - 286,200 = \$72,043.20$  Interest paid



13. Jennifer buys a house for \$300,000 with a down payment of \$50,000. She takes out a mortgage for the remainder at an interest rate of 6% (compounded monthly).

- a. Find the amount of the monthly payment needed to amortize the loan over 30 years.

$$\begin{aligned} P &= 250,000 \\ r &= .06 \\ m &= 12 \\ t &= 30 \end{aligned}$$

$$P = R \left[ \frac{1 - \left(1 + \frac{r}{m}\right)^{-mt}}{\frac{r}{m}} \right]$$

$$250,000 = R \left[ \frac{1 - \left(1 + \frac{.06}{12}\right)^{-12 \times 30}}{\frac{.06}{12}} \right]$$

$$R = \$1498.88$$

- b. Find the total amount of interest paid when the loan is amortized over 30 years.

$$1498.88 \times 12 \times 30 = 539,596.80$$

$$539,596.80 - 250,000 = \$289,596.80$$

- c. Find the part of the first payment that is interest and the part that is applied to reducing the debt.

$$\begin{aligned} P &= 250,000 \\ r &= .06 \\ t &= \frac{1}{12} \end{aligned}$$

$$I = Prt$$

$$= 250,000 \times .06 \times \frac{1}{12} = \$1250$$

Payment Number	Amount of Payment	Interest	Principal	Balance
0				250,000
1	1498.88	1250	248.88	249,751.12

$$1498.88 - 1250 = 248.88$$

1st payment  
Interest = \$1250  
Part applied to reducing the balance = \$248.88

14. For each of the following systems of equations, find all the solutions if possible. Then give a geometric interpretation of the linear system.

a.

$$\begin{array}{r} 2x+3y=19 \\ x+4y=22 \end{array}$$

$$\begin{array}{r} 2x+3y=19 \\ -2(x+4y=22) \\ \hline 2x+3y=19 \\ -2x-8y=-44 \\ \hline -5y=-25 \\ y=5 \end{array}$$

$$\begin{array}{r} 2x+3(5)=19 \\ 2x+15=19 \\ 2x=4 \\ x=2 \end{array}$$

Solution: (2, 5)

Geometric interpretation of the system:

The two lines intersect at the point (2, 5)

b.

$$\begin{array}{r} -3x+2y=12 \\ 9x-6y=-36 \end{array}$$

$$\begin{array}{r} -3x+2y=12 \\ 3(-3x+2y=12) \\ 9x-6y=-36 \\ \hline -9x+6y=36 \\ 9x-6y=-36 \\ \hline 0=0 \end{array}$$

$$\begin{array}{r} -3x+2y=12 \\ 2y=3x+12 \\ y=\frac{3}{2}x+6 \end{array}$$

Solution: (x,  $\frac{3}{2}x+6$ )

Geometric interpretation of the system:

The two equations represent the same line.

c.

$$\begin{array}{r} 2x+3y=15 \\ 6x+9y=25 \end{array}$$

$$\begin{array}{r} -3(2x+3y=15) \\ 6x+9y=25 \\ \hline -6x-9y=-45 \\ 6x+9y=25 \\ \hline 0=-20 \end{array}$$

Solution: no solution

Geometric interpretation of the system:

The two lines are parallel

15. A candy store sells cashews for \$6.50 per pound and peanuts for \$2.50 per pound. If you want to buy exactly 4 pounds of nuts for \$23.00, how many pounds of each kind should you buy?

$x =$  # of pounds of cashews  
 $y =$  # of pounds of peanuts

$$\begin{aligned} -6.5(x+y=4) \\ 6.5x+2.5y=23 \end{aligned}$$

$$\begin{array}{r} -6.5x - 6.5y = -26 \\ 6.5x + 2.5y = 23 \\ \hline -4y = -3 \\ y = \frac{3}{4} \text{ or } 0.75 \end{array}$$

3.25 lb of cashews  
 0.75 lb of peanuts

$$\begin{aligned} x + 0.75 &= 4 \\ x &= 3.25 \end{aligned}$$

16. Pat is buying toothbrushes. Small ones come 12 to a box while large ones come 8 to a box. Each box of small toothbrushes cost \$4, while each box of large toothbrushes cost \$3. If Pat uses 136 toothbrushes and they cost a total of \$49, how many boxes of each size were used?

$s =$  # of small toothbrush boxes  
 $b =$  # of big toothbrush boxes

$$\begin{aligned} 12s + 8b &= 136 \\ -3(4s + 3b &= 49) \end{aligned}$$

$$\begin{array}{r} 12s + 8b = 136 \\ -12s - 9b = -147 \\ \hline -b = -11 \\ b = 11 \end{array}$$

4 boxes of small toothbrushes  
 11 boxes of large toothbrushes

$$\begin{aligned} 4s + 3(11) &= 49 \\ 4s + 33 &= 49 \\ 4s &= 16 \\ s &= 4 \end{aligned}$$

17. A silo is made of a cone that fits perfectly on top of a cylinder. The diameter of the cone is 8 feet, the height of the cone is 3 ft, and the slant height of the cone is 5 ft. The height of the cylinder is 10 ft. Find the volume and surface area of the silo (including the base).

Volume of Cone

$$\frac{1}{3} \pi r^2 h$$

$$\frac{1}{3} \pi (4)^2 (3) = 50.3$$

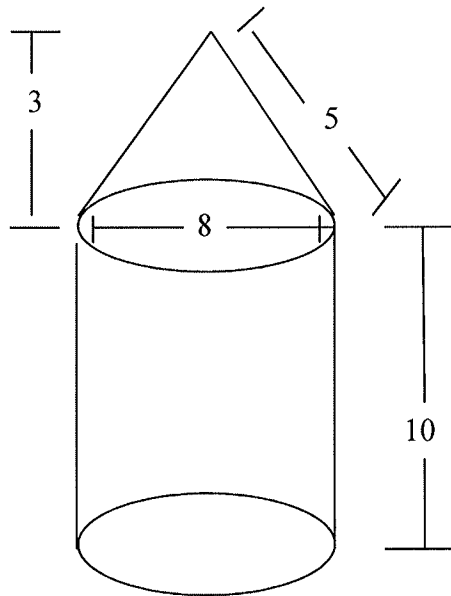
Volume of Right Cylinder

$$\pi r^2 h$$

$$\pi (4)^2 (10) = 502.7$$

Total Volume

$$\begin{array}{r} 50.3 \\ + 502.7 \\ \hline 553 \text{ ft}^3 \end{array}$$



Surface Area for cylinder  
 $2\pi r^2 + 2\pi rh$   
 (but we only want one end)

$$\begin{aligned} \pi r^2 + 2\pi rh \\ \pi (4)^2 + 2\pi (4)(10) \\ 16\pi + 80\pi = 96\pi \\ = 301.59 \end{aligned}$$

Surface Area of Cone

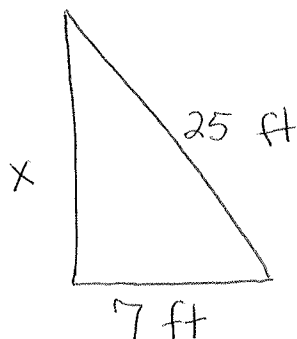
$\pi r l + \pi r^2$   
 (but we don't want the cone base)

$$\begin{aligned} \pi r l \\ \pi (4)(5) = 20\pi \\ = 62.83 \end{aligned}$$

Total Surface Area

$$\begin{array}{r} 301.59 \\ + 62.83 \\ \hline 364.42 \text{ ft}^2 \end{array}$$

18. A 25 foot ladder is placed against a building. The bottom of the ladder is 7 feet from the building. If the top of the ladder slips down 4 feet, how many feet will the bottom slide out? No, it is not 4 feet. This is a two-step problem, so draw two right triangles.



$$25^2 = x^2 + 7^2$$

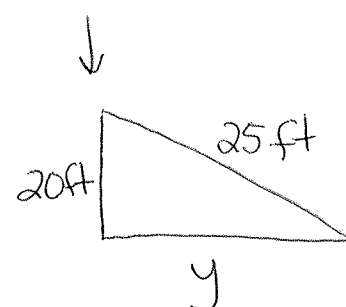
$$625 = x^2 + 49$$

$$576 = x^2$$

$$24 = x$$

The ladder reaches 24 ft up the side of the building.

down 4 ft



$$25^2 = y^2 + 20^2$$

$$625 = y^2 + 400$$

$$225 = y^2$$

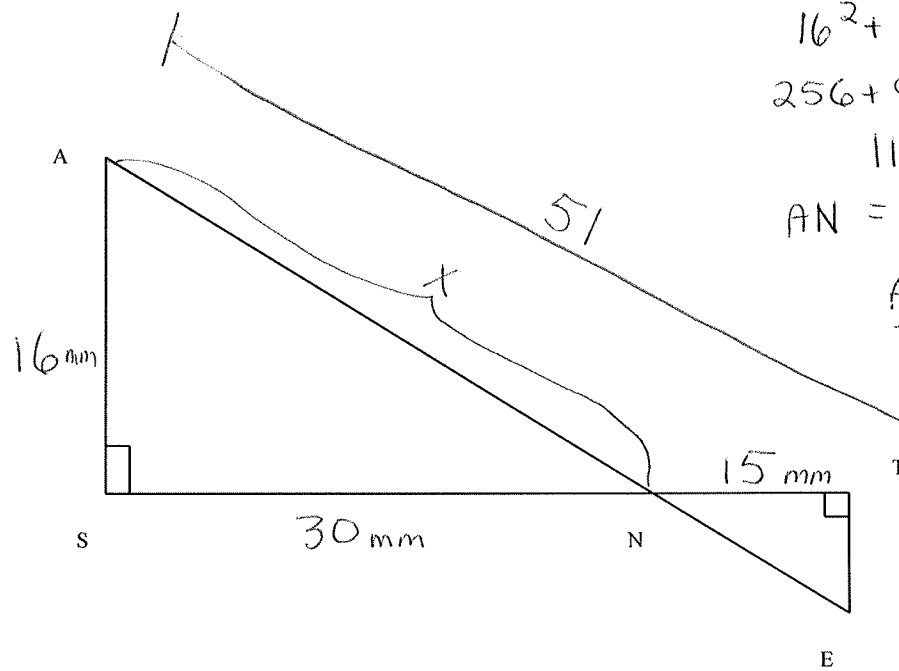
$$15 = y$$

Now it's 15 ft from the building

$$15 - 7 = 8$$

It slid out 8 feet.

19. In the figure below AE and ST intersect at N, AS = 16mm, SN = 30mm, NT = 15mm, and AE = 51mm, find NE



$$16^2 + 30^2 = x^2$$

$$256 + 900 = x^2$$

$$1156 = x^2$$

$$AN = 34 = x$$

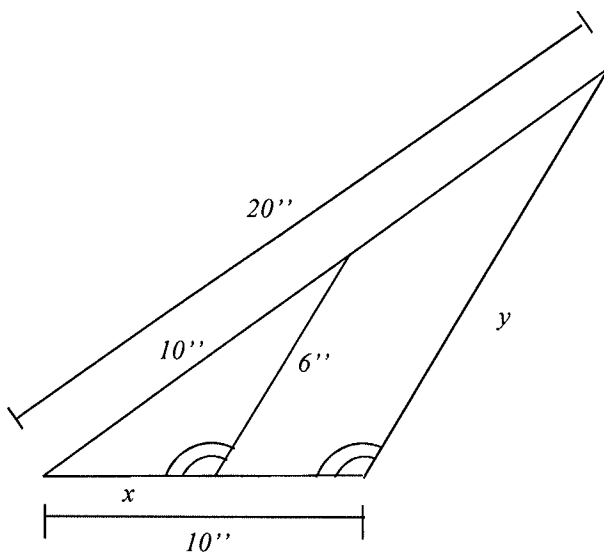
$$AN + NE = AE$$

$$34 + NE = 51$$

$$NE = 17$$

17 mm

20. Find x and y.



similar triangles

$$\frac{10}{20} = \frac{x}{10}$$

$$20x = 100$$

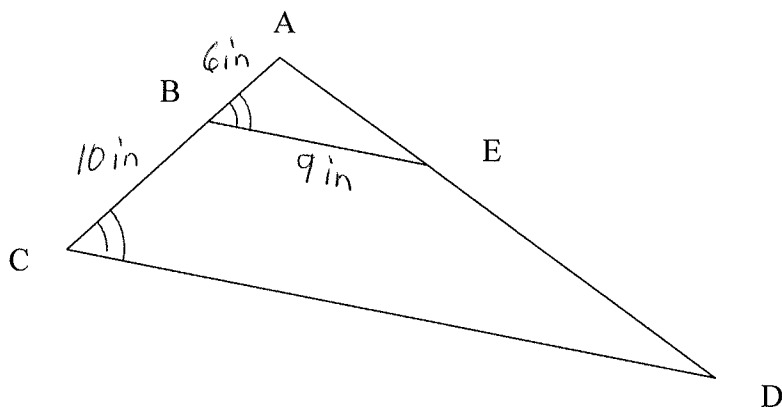
$$x = 5$$

$$\frac{10}{20} = \frac{6}{y}$$

$$10y = 120$$

$$y = 12$$

21. In the figure below,  $AB = 6$  in,  $BC = 10$  in and  $BE = 9$  in, find  $CD$



$$\frac{6}{16} = \frac{9}{CD}$$

$$6(CD) = 144$$

$$CD = 24$$

24 in

22. Find the area and perimeter of the shape shown below:

Area  
Rectangle

$$A = lw$$

$$A = 15 \times 8 = 120 \text{ ft}^2$$

Circle

$$A = \pi r^2$$

$$r = \frac{15}{2} = 7.5$$

$$A = \pi (7.5)^2$$

$$A = 176.71$$

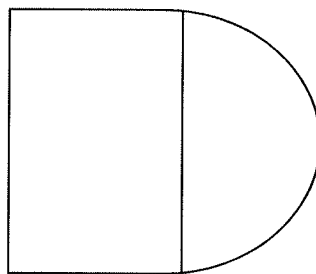
We want half of the circle

$$\frac{176.71}{2} = 88.36 \text{ ft}^2$$

$$\text{Total Area} = 120 + 88.36 = 208.36 \text{ ft}^2$$

8ft

15ft



Perimeter

Use 3 sides of the rectangle

Half the circumference of the circle

Circumference is  $2\pi r$

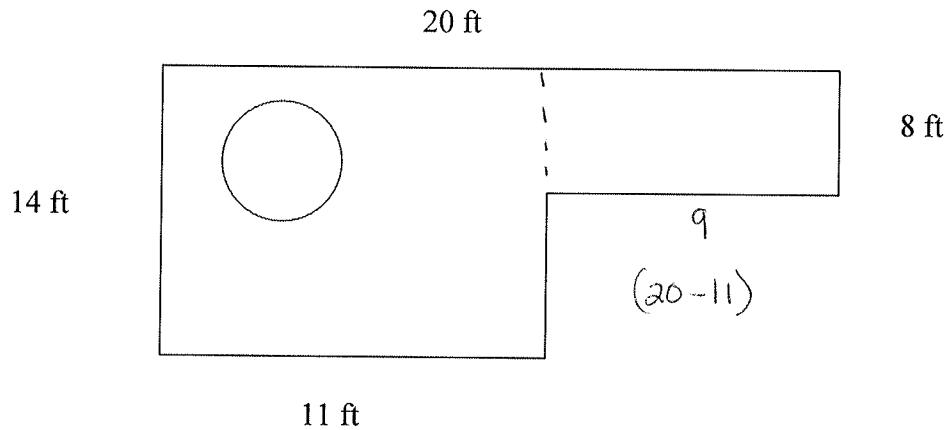
$$2\pi(7.5) = \frac{47.12}{2} =$$

Half the circumference 23.56

$$23.56 + 8 + 15 + 8 =$$

54.56 ft

23. A Cabrini intern needs to design a brick patio for her client. She needs to allow room for a hot tub with a diameter of 6 feet. What is the area that needs to be covered in brick? (round to the nearest square foot)



Area of the hot tub  
 Diameter is 6 ft so the radius is 3 ft  
 $A = \pi r^2$   
 $= \pi (3)^2$   
 $= 28.27 \text{ ft}^2$

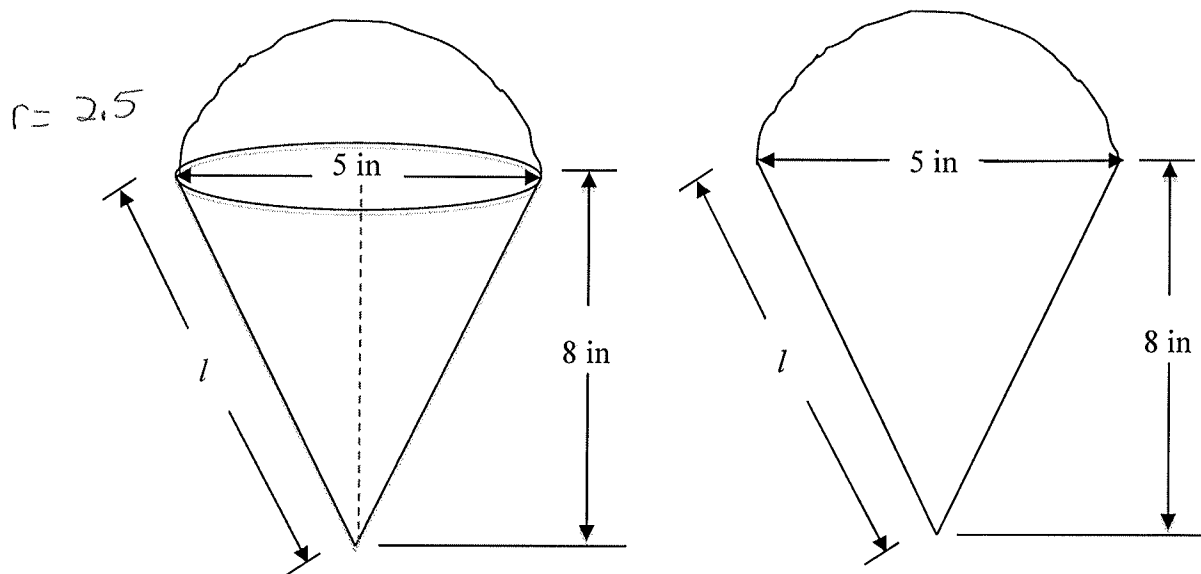
Area of rectangle with hot tub  
 $11 \times 14 = 154 \text{ ft}^2$

Area of rectangle without hot tub  
 $9 \times 8 = 72 \text{ ft}^2$

$$154 + 72 - 28.27 = 197.73$$

198 ft sq

24. Using this drawing of an ice cream cone,



a. What is the volume of ice cream and cone shown in the picture on the left?

Volume of Cone

$$\frac{1}{3} \pi r^2 h$$

$$\frac{1}{3} \pi (2.5)^2 (8)$$

$$= 52.36 \text{ in}^3$$

Volume of Sphere

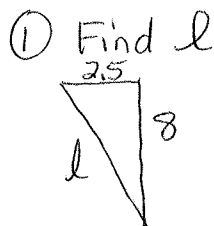
$$\frac{4}{3} \pi r^3$$

$$\frac{4}{3} \pi (2.5)^3 \quad (\text{We only have half})$$

$$= 65.45 \text{ in}^3 \quad \frac{6}{2} = 32.725$$

$$\begin{array}{r} 52.36 \\ + 32.73 \\ \hline 85.09 \text{ in}^3 \end{array}$$

b. What is the surface area of the ice cream and cone in the picture on the left?



$$8^2 + 2.5^2 = l^2$$

$$70.25 = l^2$$

$$l = 8.38$$

Surface Area of Cone  
(just the side—not the bottom)

$$= \pi r l$$

$$= \pi (2.5)(8.38)$$

$$= 65.82 \text{ in}^2$$

$$65.82 + 39.27 = 105.09 \text{ in}^2$$

Surface Area of Sphere  
(we need half of this)

$$= 4 \pi r^2$$

$$= 4 \pi (2.5)^2$$

$$= 12.5 \pi = 39.27$$

surface area of half the sphere

c. What is the area of the picture on the right?

Area of Triangle

$$\frac{1}{2} bh = \frac{1}{2} (5)(8)$$

$$= 20 \text{ in}^2$$

Area of Circle (we need half)

$$\frac{\pi r^2}{2}$$

$$\frac{\pi (2.5)^2}{2} = 3.125 \pi = 9.82 \text{ in}^2$$

$$20 + 9.82 = 29.82 \text{ in}^2$$

d. What is the perimeter of the picture on the right?

$l + l + \frac{1}{2}$  circumference of the circle

$$l + l + \frac{1}{2} (2 \pi r) = 8.38 + 8.38 + 2.5 \pi = 24.61 \text{ in}$$