

Math 130

Sample Test #3

Find dy/dx by implicit differentiation

1. $x^3 - 4xy + y^2 = -4$

2. $x^3 + y^3 = 6xy$

3. $xy + 12x + 3x^2 = 124$

4. $\sin(x + y) = y^2 \cos x$

5. $e^y \cos x = 1 + \sin(xy)$

Find derivatives of inverse trig functions

6. $y = \tan^{-1} \sqrt{x}$

7. $y = \cos^{-1}(e^{2x})$

8. $y = \sqrt{1 - x^2} \cos^{-1} x$

9. $y = \sin^{-1}(2x + 1)$

Solve the following related rates problems:

10. Rapunzle is singing in her tower far, far away when Prince Charming hears her song. "Rapunzle, Rapunzle. Let down your long hair." "Forget the hair! Use that ladder over there," she demands. Prince Charming happily grabs a 13-foot ladder conveniently leaning up against a tree nearby. He props up the ladder 5ft away, thinking it would reach the small window. While he was climbing, his ladder starts to slide down the wall at a rate of 1 ft. per minute. How fast is the base of the ladder moving when it hits the ground? Draw a diagram and show all work.

11. Jason Bourne is on the run again. He's racing down the street in his Mini-Cooper, not realizing that the police are coming up on the opposite street. Bourne is traveling east at 80 miles per hour while the police are racing north at 100 miles per hour. Both cars are approaching a junction in the street rather quickly. If Bourne started 7 miles away from the intersection and the police started 24 miles away, at what rate are the cars approaching each other at the intersection? Draw a diagram and show all work.

12. Harry Problem looks out of his window on Pythagorean Drive and notices the lights going out along his street one by one. He becomes curious and walks careful out of his house and towards the 12 foot high lamppost. If he is walking at the rate of 3 miles per hour, at what rate is he approaching when he is starting 16 feet away from the pole? Draw a diagram and show all work.

13. Deebo the clown is at a carnival and blowing up a huge balloon with her automatic air pump (she really wouldn't have enough air in her lungs for this one). The spherical balloon is being inflated at the rate of 20 cubic feet per minute. At the instant when the radius is 15 feet, at what rate is the radius increasing?

14. So let's say that bacteria that are now living on your door handle into your dorm building triple every hour. It starts with 400 bacteria. Find an expression for the number n of bacteria and t hours and use it to estimate the rate of growth of the bacteria population after 2.5 hours. (Look at Example 6 on pg. 226 to give you an idea of how to figure out the formula).

Find the critical numbers of the functions:

15. $f(x) = x^3 - 3x^2 + 1$ on the interval $[-1/2, 4]$

16. $f(x) = 2t^3 - 7t^2 + 22$

Find the absolute maximum and absolute minimum values of f on the interval $[-3, 2]$

17. $f(x) = (x^2 - 1)^3$ on the interval $[-1, 2]$

18. $f(x) = x^3 - 3x + 1$ on the interval $[0, 3]$

Find where the function is increasing and where it is decreasing

19. $f(x) = -4x^3 - 12x^2 + 2$

20. $f(x) = x^3 + 3$

At what value of x does the function change from decreasing to increasing?

21. $f(x) = x^2 - 6x + 7$

For the function $y = x^4 - 8x^3$, find:

22. The point(s) of inflection

23. Where the graph is concave upward

24. Where the graph is concave downward

For the function $f(x) = 2x^4 - 6x^2$

25. Graph the function

26. Find all the maximums and minimums of the graph

Find the antiderivative

27. $f(x) = 4x^3 + 6x^2 - x + 12$

28. $f(x) = 12x^{-3}$

Solve the following problems:

29. It's almost the end of the semester and you're finally starting to pack everything up. You want to make a box with an open top from a square piece of cardboard, 3 inches wide, by cutting out a square from each corner and bending up the sides. Find the largest volume such a box can have. List the dimensions and the volume. Show a diagram with all your work.
30. You decided that you wonderful math tutor you made this sample test deserved a little something special. Therefore you grab a piece of colorful wire 20 inches long to cut and make into a rectangular to frame a picture on a scrapbook page. What dimensions should be chosen so that the area of the rectangle enclosed is maximal?
31. Your dad has recently shown an interest in farming oddly enough. He has 6,000 feet of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field with the largest area?

The position of a particle is given by the equation $f(t) = t^3 - 12t^2 + 36t$.

32. Find the velocity at time t .
33. What is the velocity after 3 seconds?
34. When is the particle at rest?
35. When is the particle moving in the positive direction?
36. Find the total distance traveled during the first 8 seconds.

37. Find the acceleration at time t .
38. What is the acceleration after 3 seconds?
39. Find the jerk at time t .
40. What is the jerk after 2 seconds?

State True or False

41. ____ If f is a continuous function over the Real numbers and $f'(c)=0$ or $f'(c)$ does not exist, then f has a local max or min at c .
42. ____ A critical number of a function f is a number c in the domain of f , such that $f'(c)=0$ or $f'(c)$ does not exist.
43. ____ If f is continuous on $[a, b]$ then f will always attain an absolute max value $f(c)$ and an absolute min value $f(d)$ at some numbers c and d in $[a, b]$.
44. ____ If $f'(x) > 0$ for $2 < x < 5$ then x is decreasing on $(2, 5)$.
45. ____ A point P on a curve is called an inflection point if the curve changes from concave upward to concave downward or from concave downward to concave upward at P .

Differentiate the functions:

46. $f(x) = \ln \frac{(3x+2)^4}{(2x-5)^3}$

47. $f(x) = \sin(\ln x)$

48. $y = \frac{\ln x}{x^2}$

49. $y = \ln(e^{-x} + x e^{-x})$

50. $f(x) = \ln(\sin x^2)$

Use Logarithmic differentiation to find the derivative:

51. $y = \frac{(6x+1)^5}{(4x^2+2)^3}$

52. $y = \frac{(2x+1)^4(x^3-3)^5}{\sqrt{3x^5-9}}$

53. $y = (\sin x)^{\ln x}$

54. $y = x^{\sin x}$

Given: $f(x) = x^3 + 3x^2 + 4$

55. Find all critical points

56. Find the intervals of increase and decrease

57. Find local maximum and minimum values

58. Find all points of inflection

59. Find where the graph is concave upward and concave downward

60. Sketch the graph and label all max, min, points of inflection, etc.

Find the first, second, and third derivatives of the following:

61. $y = x(x^2 - 3)(x^3 + 1)$

62. $y = (x + 2)^2 - 5x^3$

63. $y = \sin^3 x$

64. $y = x^6 + \frac{2}{x^2} - \sin x$

Solve the following:

65. Suppose \$5000 is put into an account that pays 4% compounded continuously. How much will be in the account after 3 years?