Sample Problems from Calculus I

1.) For $f(x) = 10 - 5x - x^2$, find:

a. $f(-2) =$ _____

b. $f(2x) =$ _____

c. $f(x+h) =$ _____

d. $5f(x) =$ _____

e. $f(1+h) =$ _____

2.) Be sure to read each part carefully.

a. Find the slope of the line that contains the points (5,5) and (-3,1) m = _____

b. Find the equation of the line with slope = 3 which passes through (3,2). Put your answer in the slope-intercept form $(y = mx + b)$. _____

c. Find the equation of the line which passes through the points (2,-4) and (-1,2)._____

3.) Evaluate the limit, if it exists:

a. $\lim_{x\to 2}\left(\frac{9}{(x-2)}\right)=$

b. $\lim_{x\to -2}(6x^2+4x-20)=$

c. $\lim_{h\to 0}\left(\frac{(h-4)^2-16}{h}\right)=$

d. $\lim_{x\to -3}\left(\frac{(x^2-2x-15)}{(x+3)}\right)=$

4.) Let: $f(x)=\begin{cases}4x^2+3x-2, if \ldots\ldots x<1\\ 5+x, if \ldots\ldots\ldots\ldots x\geq 1\end{cases}$

Find:

a. $\lim_{x\to 1-} f(x) =$ _____

b. $\lim_{x\to 1+} f(x) =$ _____

c. Does $\lim_{x\to 1} f(x)$ exist? _____

5.) Let: $f(x) = \begin{cases} 8 - x, if \ldots\ldots\ldots x \le 4 \\ x^2 - 3x, if \ldots\ldots\ldots x > 4 \end{cases}$

a. Is $f(x)$ continuous at 4? _____

b. Why or Why Not? _____

6.) Given $f(x) = 4x^2 + 8x$ and $g(x) = 4x - 6$, find:

a. $(f \circ g)(1) =$ _____

b. $(g \circ f)(1) =$ _____

c. $(g \circ g)(1) =$ _____

7.) If a ball is thrown into the air with a velocity of 58 m/s, its height in meters after *t* seconds is given by $h = 62t - .79t^2$. Find the average velocity over the time interval [1,3]. _____

8.) The point (0,2) lies on a curve $f(x) = 2 - 3x^3$. If Q is the point $(1, f(1))$, find the slope of the secant line PQ. _____

9.) If $f(x) = \dfrac{(x^2 - x - 12)}{(x+3)}$, for $x \neq -3$, and $f(x)$ is continuous at –3, what must $f(-3)$ equal? _____

10.) Use the definition of derivative $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ to find the derivative of $f(x) = 6x^2 - 4x + 3$

11.) Differentiate each of the following functions:

a. $f(x) = 2x^6 - 3x^{-2} + 2x\sqrt{x} + 4x + 26$

b. $f(t) = t^{\frac{3}{4}} - \dfrac{4t}{\sqrt{t}}$

c. $h(x) = (x^2 - x^3)(x + 2x^2)$

d. $f(x) = \dfrac{x^2 - 8x + 16}{x - 4}$

e. $y = \dfrac{4t - 3}{2t^2 + 4t - 1}$

f. $y = x\sqrt{x} + \dfrac{1}{x^3\sqrt{x}} + \dfrac{1}{x^7}$

g. $f(x) = x^{\Pi} + x^2 + 5$

h. $g(s) = as^2 - b^2 - c$

12.) Find the equation of the tangent line to the function $y = \frac{4x}{2x+4}$ at the point (-1,-2). Write your answer in the $y = mx + b$ form.

13.) Find the points on the curve $y = x^3 + 4x^2 + 4x + 10$ where the tangent is horizontal.

14.) A particle moves according to a law of motion $s = f(t)$, $t \geq 0$, where *t* is measured in seconds and *s* in feet.

a. Find the velocity at time t = _____

b. What is the velocity at time t =1? _____

c. What is the speed at 1 second? _____

d. When is the particle at rest? _____

e. When is the particle moving in the positive direction? _____

15.) Suppose that the cost in dollars, for a company to produce x pairs of a new line of jeans is $c(x) = 2000 + 3x + .02x^2 + .0001x^3$. Find:

a. The marginal cost function _____

b. Find $c'(102)$ _____. What does it predict? _____

c. What will it cost to produce the 102nd pair? _____

16.) Find $\frac{dy}{dx}$ (i.e. y') by implicit differentiation.

a. $xy = 25$

b. $x^2 + 3xy + y^2 = 15$

c. $\cos(xy) = 1 - x^2$

d. $x + \frac{1}{y} = 5$

e. $x^2 + y^3 = 12$

17.) Find an equation of the tangent line to the graph of each equation at the specified point.

a. $x^2 + y^2 = 13$ at (-2,3)

b. $x^2 + 2xy - y^2 + x = 2$ at (1,2)

18.) Find f', f'', f''', f''''

a. $f(x) = x^5 - 5x^3 + x + 12$

b. $f(x) = \frac{1}{4}x^8 - \frac{1}{2}x^6 - x^2 + 2$

c. $f(x) = \frac{-2}{x^2}$

d. $f(x) = \frac{4}{\sqrt{x}}$

19.) Find the absolute maximum and absolute minimum values of f on the given interval

a. $f(x) = 5 + 10x - x^2$ on [-3,3]

b. $f(x) = x^3 - 3x$

c. $f(x) = 2x^3 - 3x^2 - 36x + 4$ on [-4,4]

20.) Find critical points and where the function is increasing & decreasing

a. $f(x) = x^3 + 3x^2 + 1$

b. $f(x) = x^3 + 35x^2 - 125x - 9{,}375$

c. $f(x) = x^5 - 5x^4 + 100$

d. $f(t) = (t+1)^2(t-5)$

21.) For $h(x) = 3x^5 - 5x^3 + 3$

a. Find the intervals of increase or decrease

b. Find the local maximum and minimum values

c. Find the intervals of concavity & the inflection points

22.) Use your rules for logarithms to solve for x:

a. $\log_4 x = 3$

b. $\log_3(6x-15) = 2$

c. $\log_9 x = \frac{1}{2}$

d. $\log_{\frac{1}{4}} 256 = x$

e. $\log_x 64 = \frac{3}{2}$

f. $\log_b x = \frac{2}{3}\log_b 125 + \frac{1}{2}\log_b 16 - \log_b 4$

g. $\log_b x + \log_b(x+14) = \log_b 45$

h. $2\log_3(x+4) - \log_3 9 = \log_3 2$

23.) Add finding the derivative by taking the ln of each side problems.

a. y= $\frac{(3x+1)^4}{(5x^2+4)^7}$

b. $y = x^{\cos 3x}$

c. $y = \sqrt{x^3 \sec 5x}$

d. $y = (\sin x)^{6x}$

24.) Find the derivatives of inverse trigonometric functions:

a. $y = \sqrt{x^2 - 1}\sec^{-1} x$

b. $y = \sin^{-1}(3x^2 + 4x)$

c. $y = \tan^{-1}\sqrt{x^3}$

d. $y = \sqrt{\tan^{-1} x}$

25.) Obi-Wan Answer is on top of a 10-foot ladder leaning up against the back wall of an apartment building as he secretively spies on the evil enemy Darth Variable who is only 6 ft. away . All of a sudden, the “force” that was holding up his ladder fails and starts to slide down the wall at a rate of 4 ft. per minute. How fast is the base of the ladder moving when it hits Darth Variable? Draw a diagram and show all work.

26.) There is a race for the Aztec gold treasure between William TurnAround and Capt. Jack SpareMe and Captain BarBossy. William and Jack are traveling from the west due east at 15 mi/h and BarBossy is making his way from the south due north at 10 mi/h, to reach the same location. If William and Jack started 40 miles from the destination and BarBossy started 30 miles away, at what rate are the ships approaching each other as they pursue the treasure? Draw a diagram and show all work.

27.)Harry Problem looks out of his window on Pythagorean Drive and notices the lights going out along his street one by one. He becomes curious and walks careful out of his house and towards the 12 foot high lamppost. If he is walking at the rate of 3 miles per hour, at what rate is he approaching when he is starting 100 feet away from the pole? Draw a diagram and show all work.

28.) The position of a particle is given by the equation $s = f(t) = t^3 - 12t^2 + 4t$ where t is measured in seconds and s in meters.
    a. Find the position after 7 seconds

    b. Find the velocity at time t

    c. Find the acceleration at time t

    d. What is the acceleration after 7 seconds?

    e. What is the jerk at time t?

    f. What is the jerk after 3 seconds?