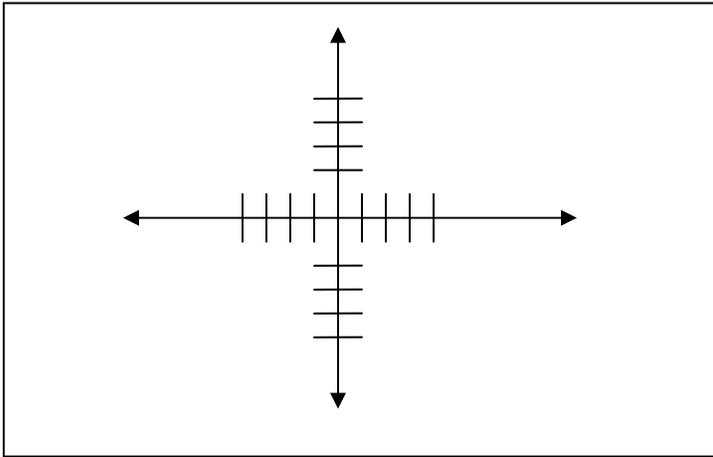


Math 113 - Review for Exam I

Section 1.1 – Cartesian Coordinate System, Slope, & Equation of a Line

- (1.) **Rectangular or Cartesian Coordinate System** – You should be able to label the quadrants in the rectangular or Cartesian coordinate system. You should also be able to graph a given point. The origin is defined as the point (0,0).

Sample Problems: Label the quadrants, x-axis, & y-axis in the graph shown below and plot the points (3,1) (-2,2) (-1,-1) (2,4) & (0,0).



- (2.) **General Equation of a Line**

$$Ax + By = C$$

x-Intercept – Is the point on a line that intercepts (crosses) the x-axis. To find the x-intercept let $y = 0$ and solve for x .

y-Intercept – Is the point on a line that intercepts (crosses) the y-axis. To find the y-intercept let $x=0$ and solve for y .

Example: Given the line $5x + 2y = 10$ find the x-intercept and the y-intercept.

x-intercept: $5x + 2(0) = 10$

$$5x = 10$$

$$x=2$$

x-intercept = (2,0)

y-intercept: $5(0) + 2y = 10$

$$2y = 10$$

$$y=5$$

y-intercept = (0,5)

Once you have defined two points on a line you can graph the line.
Remember: The x-intercept and y-intercept are points on a line and all points should be defined by both their x and y values.

Sample Problems:

2.1 Find the x-intercept and y-intercept for the following lines:

a. $3x + 5y = 30$

b. $2x + 4y = 16$

c. $7x + 2y = 14$

d. $3x + 2y = 8$

Now graph and label the line in 2.1.d using the 2 points you defined.

2.2 Complete the table of values for the given lines:

$3x + 2y = 24$

x	0		4		9
y		0		3	

$2x - y = 8$

x	0		1	5	
y		0			6

(3.) Slope of a Line

The slope of a line is defined as the average rate of change of y with respect to x.

$$\text{Slope} = m = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example: Find the slope of a line that contains the points (4,2) and (6,8)

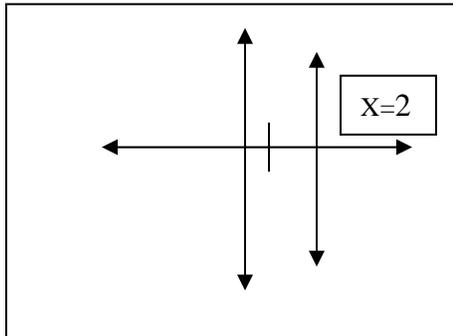
$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 2}{6 - 4} = \frac{6}{2} = 3$$

The slope in our example is positive, this means as y increases, x increases and as y decreases, x decreases. Specifically for this example for every +3 unit change in y there is a +1 unit change in x . For instance if the y value on this line went from 8 to 11 (a 3 unit increase) then the x value would go from 6 to 7 (a 1 unit increase) OR if the y value decreased from 2 to -1 (a 3 unit decrease) then the x value would decrease from 4 to 3 (a 1 unit decrease).

If the slope is negative this means as y increases, x decrease of as y decreases, x increases.

You should know how to calculate the slope of a line given two points on the line. You should also know how to graph a line if you are given a point on the line and the slope of the line. You should also what direction a line with a positive slope slants and what direction a line with a negative slope slants.

Vertical Line: A vertical line graphed in the rectangular coordinate system looks like this:



In a vertical line, the x value stays constant. For the line above ($x=2$) no matter what the y -value is. The slope of a vertical line is undefined.

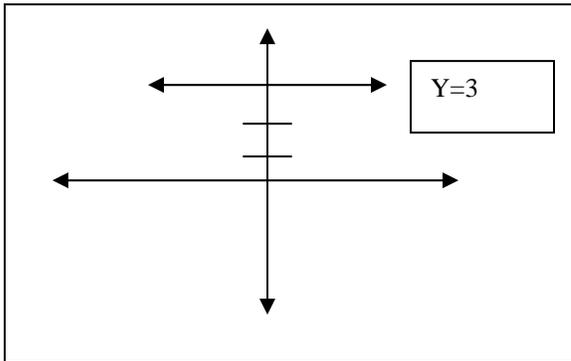
For example: The slope of a vertical line containing the points (2,4) and (2,-1) is calculated as:

$$m = \frac{4 - (-1)}{2 - 2} = \frac{5}{0} \quad \text{Can't divide by zero, so slope is undefined}$$

The general equation for a vertical line is $x = a$, the equation of the line above is

$$x = 2.$$

Horizontal Line: A horizontal line graphed in the rectangular coordinate system looks like this:



In a horizontal line, the y value stays constant. For the line above ($y=3$) no matter what the x -value is. The slope of a horizontal line is ZERO.

For example: The slope of a horizontal line containing the points $(2,3)$ & $(-2,3)$ is calculated as:

$$m = \frac{3-3}{2-(-2)} = \frac{0}{4} = 0$$

The general equation for a horizontal line is $y = b$, the equation of the line above is $y = 3$.

You should know how to define the equation of a vertical or horizontal line either (1) given that the line is horizontal or vertical and a point on the line or (2) given the slope of the line (undefined or 0) and a point on that line.

Sample Problems:

3.1 Find the slope of the line containing the points $(2,1)$ & $(4,6)$. Use the slope of this line to find another point on the line and graph the line. LABEL YOUR POINTS.

3.2 Find the slope of the line containing the points $(4,2)$ & $(7,8)$.

3.3 Find the slope of the line containing the points (2,3) & (2,7). What type of line is this? What is the equation of this line?

3.4 Find the slope of the line containing the points (3,4) & (6,4). What type of line is this? What is the equation of this line?

3.5 What is the slope of a vertical line?

3.6 What is the slope of a horizontal line?

3.7 What is the equation of the line with a slope = 0 and containing the point (3,4)?

3.8 What is the equation of a vertical line containing the point (1,3)?

(4.) Finding the Equation of a Line

To find the equation of a line given the slope of the line and a point on the line you should use the following equation:

The Point-Slope Equation:

$$y - y_1 = m(x - x_1)$$

where: m = slope and (x_1, y_1) is the given point.

For example: Find the equation of a line with a slope = 3 and containing the point (1,4)

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 3(x - 1)$$

$$y - 4 = 3x - 3$$

$$y = 3x + 1$$

Using the **Point-Slope Equation** and solving for y as shown in the example above gives you the equation of a line in the **Slope-Intercept Form**.

$$y = mx + b$$

where:

$m = \text{slope}$

$b = y \text{ value of the } y\text{-intercept (}y\text{-intercept} = (0,b)$

If you are given two points on the line, then you first need to find the slope of the line and then use the **Point-Slope Equation** and ONE of the given points to find the equation of the line.

Sample Problems:

Find the equation of the lines in **Slope-Intercept Form**:

4.1 With a slope = 2 and the point (2,3)

4.2 With a slope = $\frac{3}{2}$ and the point (4,6)

4.3 Containing the points (2,4) & (4,2)

4.4 Containing the points (-2, -1) and (2, -6)

Find the equation of the lines in **General Equation Form**:

4.5 With a slope = -4 and containing the point (2,-3)

4.6 With a slope = $\frac{4}{3}$ and containing the point (-6,2)

4.7 Containing the points (1,3) & (-2,-4)

4.8 Containing the points (3,2) & (3,0)

(5.) Word Problems

The word problems in section 1.1 are asking you to find the equation of a line given two points on that line or one point on that line and the slope. The key to solving the word problems is being able to pull out the required information.

A specific type of word problem asks you to find the equation that linearly relates cost and number of things produced. You should remember that cost equations have a fixed cost and a variable cost. The variable cost changes and depends on the number of things you produce.

$$\begin{aligned} \text{Cost} &= \text{Variable Cost} + \text{Fixed Cost} \\ \text{OR using the Slope-Intercept Form} \\ y (\text{cost}) &= mx (\text{variable cost}) + b (\text{fixed cost}) \end{aligned}$$

Example: The cost of producing ipods can be linearly related to the number of ipods produced. If 100 ipods are produced the cost is 102,500 and if 2000 ipods are produce the cost is \$150,00. Find the equation for the cost of producing ipods.

Let y = cost producing ipods and x =the number of ipods produced. You are given two order pairs in the problem: (100,102500) and (2000, 150000) – use these ordered pairs to find the equation.

First find the slope, then use the slope and one pointing the Point-Slope Equation to find the equation of the line:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{150000 - 102500}{2000 - 100} = \frac{47500}{1900} = 25$$

$$y - y_1 = m(x - x_1)$$

$$y - 102500 = 25(x - 100)$$

$$y - 102500 = 25x - 2500$$

$$y = 25x + 100000$$

How much would it cost to produce 5000 ipods? (plug-in 5000 for x and solve for y)

How many ipods can you produce with \$162,500? (plug-in 162500 for y and solve for x)

Some problems in this section will tell you the fixed (\$amount) and variable costs(\$/per item) , to get your cost equation pull these out and put in the $y = mx + b$ format.

Sample Problems:

- 5.1) A doughnut shop has a fixed cost of \$124 per day and a variable cost of \$0.12 per doughnut. Find the total daily cost of producing x doughnuts (the total cost equation). How many doughnuts can be produced for a total daily cost of \$3986

How much is the total daily cost to make 450 doughnuts?

- 5.2) A plant can manufacture 100 golf clubs per day for a total daily cost of \$7,800 and 120 golf clubs per day for a total daily cost of \$8,400. Assuming that the daily cost and production are linearly related, find the total daily cost of producing x golf clubs.

How much would it cost to produce 250 golf clubs per day?

- 5.3) A small company manufactures picnic tables. The daily fixed cost is \$1,200 and the variable cost is \$45 per table. Find the total daily cost of producing x picnic tables.

How many picnic tables can be produced for a total daily cost of \$4,800?

To get more practice with word problems like these you should review problems 73-84 on page 17.

Section 1.2 – Parallel Lines, Perpendicular Lines, Identical Lines, & Points of Intersection

In section 1.2 you compare lines to determine if they are parallel, intersecting, or coincident (identical).

When comparing two lines to determine if they are the same line, or if they are parallel or if they intersect you should first put the equation for the line in the **Slope-Intercept form** ($y = mx + b$), and then compare their slopes (m) and y-intercepts (b).

Parallel Lines: Do not have any points in common and have the same slope. ($m_1 = m_2$)

Intersecting Lines: Have ONE point in common and do not have the same slope. A specific type of intersecting lines is **Perpendicular Lines**. Perpendicular lines have slopes that are negative reciprocals of each other. If L1(line 1) is perpendicular to L2 (line 2) then $m_1 * m_2 = -1$. Or if $m_1 = 3$, then $m_2 = -1/3$.

Coincident or Identical Lines: Have an infinite (all) number of points in common. They have the same slope and same y-intercept. (i.e. $m_1 = m_2$ and $b_1 = b_2$).

You should be able to determine if lines are parallel, intersecting, perpendicular, or identical.

Example: Determine if the following lines are parallel, perpendicular, or identical:

$$L1: 3x + 4y = 9 \text{ and } L2: 3y - 4x = 6$$

First put lines in slope-intercept form:

$$3x + 4y = 9$$

$$3y - 4x = 6$$

$$4y = -3x + 9$$

$$3y = 4x + 6$$

$$y = -3/4 x + 9/4$$

$$y = 4/3 x + 2$$

Since the slopes are negative reciprocals of each other these lines are perpendicular.

Finding the Point of Intersection:

When two lines intersect they have one point in common – this is their point of intersection. To find the point of intersection you need to first put the equations of the lines in the slope-intercept form, then solve for x. This x value is the x value of your point of intersection. Plug this x value into one of the equations of the lines to find your y value. The point of intersection satisfies both equations.

Example: Find the point of intersection for the following two lines:

$$L1: 12x - 3y = 15 \text{ and } L2: y + 2x = 7.$$

First put the line in slope-intercept form:

$$12x - 3y = 15$$

$$y + 2x = 7$$

$$-3y = -12x + 15$$

$$y = -2x + 7$$

$$y = 4x - 5$$

Now, set the equations equal to each other and solve for x:

$$4x - 5 = -2x + 7$$

$$6x = 12$$

$$x = 2$$

Plug this x value into one equation and solve for y

$$y = 4x - 5 = 4(2) - 5 = 3$$

Check to make sure that the point (2,3) satisfies both equations:

$$y = 4x - 5$$

$$3 = 4(2) - 5$$

$$3 = 8 - 5$$

$$y = -2x + 7$$

$$3 = -2(2) + 7$$

$$3 = -4 + 7$$

Point (2,3) satisfies both equations, so this is the point of intersection.

You should also know how to find the equation of a line if you are told the line is either parallel or perpendicular to a given line and contains a given point.

Example: Find the equation of a line that contains the point (6,2) and is perpendicular to the line $6x - 4y = 24$.

First find the slope of the given line by putting the equation of that line in Slope-Intercept form.

$$6x - 4y = 24$$

$$-4y = -6x + 24$$

$$y = \frac{3}{2}x - 6$$

The slope of the given line is $\frac{3}{2}$, the slope of a line perpendicular to this line would be

$-\frac{2}{3}$. We now have the slope of our line and one point (6,2) and we can find the equation of the line.

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-2}{3}(x - 6)$$

$$y - 2 = \frac{-2x}{3} + 4$$

$$y = \frac{-2x}{3} + 6$$

Sample Problems:

Determine if the following lines are parallel, coincident, or if they intersect. If they intersect, then find the point of intersection.

(1.1) $2y - 4x = 8$ $y = 2x + 4$

(1.2) $y - 3x = 6$ $3y = 15 - x$

(1.3) $2y + 3x = 13$ $4x - 2y = 8$

(1.4) $x + y = 5$ $3y - 2x = 10$

(1.5) $2y - 4x = 6$ $y - 3 = 2x$

(1.6) Find the equation of the line that is perpendicular to the line $2x + 6$ and contains the point $(2,6)$

(1.7) Find the equation of the line that is parallel to the line $3x + 5$ and contains the point $(1,2)$

Section 1.3 – Linear Equation Applications – Predicting Costs, Break-Even Point, & Mixture Problems.

In section 1.3 you apply what you learned in sections 1.1 and 1.2 to solve word problems involving predicting costs, break-even points, supply and demand, & mixture problems. You should review the sample problems and homework problems in section 1.3.

Predicting Costs – These problems require you to *find the equation* of a line and then use that equation to predict costs. Within the word problem you will be given two points on the line. Set your x and y values, find your slope, and then use your slope and one point to find the equation of the line – i.e. the cost function. Given a total cost (y value) use your equation to solve for x , OR given an x value solve for your total costs (y).

Break Even Problems – Break-Even problems are problems that require you to find the *point of intersection* of the linear equation that describes costs, $C(x)$ and the linear equation that describes revenue, $R(x)$. The break-even point is where the costs = revenue. To find the break even point set $C(x) = R(x)$, solve for x and then plug your x value (number of items produced) into one of the equations and solve for y (costs). At the break-even point the company is not making money but they are losing money either. When $\text{Costs} > \text{Revenue}$ the company is losing money and when $\text{Revenue} > \text{Costs}$ the company is making money. To find the Profit equation $P(x) = R(x) - C(x)$, make sure to simplify your expression. You will also need to know how to graph the cost function $C(x)$ and the revenue function $R(x)$.

Supply and Demand – For these types of problems you will be given two points on the price-demand equation (quantity, price) and will need to find the price-demand equation. Remember buyers (consumers) demand. You will also be given two points on the price-supply equation (quantity, price) and will need to find the price-supply equation. Remember manufacturers supply. You might be asked to find the demand or supply quantity for a given price or the demand or supply price for a given quantity – to solve these plug-in what you know, solve for what you don't. BE CAREFUL TO USE THE RIGHT EQUATION! The most popular question for Supply and Demand problems is to find the market equilibrium – the *point of intersection* between the price-demand equation and the price-supply equation. To do this set your equations equal to each other, solve for the equilibrium quantity and then plug this value into one of the equations to solve for the equilibrium price. At market equilibrium both consumers and suppliers are happy!

Mixture Problems: Mixture problems will involve two unknowns and two equations. You will have to use the given equations to solve for the unknowns. To do this you will follow the same steps you use to find the *point of intersection* between two linear equations. You can do this by putting both equations into the Slope-Intercept form ($y = mx + b$), setting the equations equal to each other, and then solving for x , then plug in the x value to solve for y .

AS WITH ALL WORD PROBLEMS MAKE SURE TO LABEL YOUR ANSWERS!

Sample Problems:

1. Yearly car insurance rates have risen in the last few years. Assume that the cost for insuring a car is linearly related to the year. The rate for a small car in 1997 was \$678. In 1999 the rate for a small car was \$798. Find the linear equation relating cost to insure to year. Predict what the cost will be to insure a car in 2008.
2. Producing x units of tacos costs $C(x) = 5x + 20$; revenue is $R(x) = 15x$, where $C(x)$ and $R(x)$ are in dollars.
 - a. What is the break-even quantity (i.e. x value)
 - b. What is the profit from 100 units?
 - c. How many units will produce a profit of \$500?
3. To produce x units of a religious medal costs $C(x) = 12x + 29$. The revenue is $R(x) = 25x$. Both $C(x)$ and $R(x)$ are in dollars.
 - a. Find the break-even quantity (x value)
 - b. Find the profit from 250 units.
 - c. Find the number of units that must be produced for a profit of \$130.
4. A candy store sells a candy bar mixture of milk chocolate bars and dark chocolate bars. Each bag of the candy bar mixture sells for \$11.50 and holds 50 candy bars. If the milk chocolate bar each cost \$.20 to produce and each dark chocolate bar costs \$.30 to produce, how many milk chocolate bars and dark chocolate bars should be in each bag to break-even. Would you increase or decrease the number of milk chocolate bars in order to obtain a profit?
5. A bank has outstanding loans worth \$10,000. Some were loan at 10% and some were loaned at 7%. If the income from these loans was \$820, how much was loaned at 10%?
6. A manufacturer produces mugs at a daily cost of \$2 per item. It sells them for \$5 per item. The daily operational cost is \$500. Write a cost function, the revenue function, and the profit function. Find the break-even point. If the company

made \$1000, how many items did they sell? If the company sold 110 items how much profit did they make?

7. The Academic T-Shirt Company did a cost study and found that it costs \$1400 to produce 600 "I Love Math" T-shirts. The total cost is \$1600 for a volume of 700 T-shirts.
 - a. Determine the linear cost-volume function.
 - b. What is the fixed cost?
 - c. What is the unit cost?

8. The Computer Shop sells computers. The shop has a fixed cost of \$1500 per week. Its average cost per computer is \$649 each, and the average selling price is \$899 each.
 - a. Write the linear cost function
 - b. Write the linear revenue function
 - c. Find the cost of selling 37 computers per week
 - d. Find the revenue from selling 37 computers
 - e. Find the break-even point.

9. If a product has a selling price of \$518, consumers are willing to buy 90 units of it. If the price is \$265, they will buy 200 units. A manufacturer will produce 100 units of the product if the selling price is \$250 or 210 units if the selling price is \$382.
 - a. Find the demand equation.
 - b. Find the supply equation.
 - c. At what price would manufacturers produce 150 units of the product?
 - d. At what price would consumers buy 175 units of the product?
 - e. Find the market equilibrium point (both price and quantity)